Final Report

GENERALIZED EQUILIBRIUM MODELING: THE METHODOLOGY OF THE SRI-GULF ENERGY MODEL

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PREFACE

The purpose of this report is to provide documentation of the generalized equilibrium modeling methodology underlying the SRI-Gulf Energy Model. The Federal Energy Administration, at the suggestion of Dr. William Hogan, supported the development of this documentation in order to (1) assess the quality of model output as it is used for government energy policy analysis, and (2) determine the role of the methodology in future government energy model development. This report supplements other publications that describe the SRI-Gulf model applications and input data [1, 2, 3, 4, 5, 6]. Since these other reports are detailed, comprehensive, and publicly available, this report focuses entirely on the philosophical, mathematical, and computational aspects of the methodology. We hope that this report will encourage critique of the model and its methodology by model-user's and the academic community, and will facilitate further research.

The development of the SRI-Gulf model and the generalized equilibrium modeling methodology has been a ten-year effort by many organizations and individuals. The generalized equilibrium methodology and its practical implementation could not have been accomplished without the creative contributions of Dean Boyd, Dale Nesbitt, Warner North, and Bill Rousseau. Financial support for this work was provided by the Gulf Oil Corporation, Stanford Research Institute, Council on Environmental Quality, National Science Foundation, Energy Research and Development Administration, Lawrence Livermore Laboratory, and the Electric Power Research Institute. Many individuals from these organizations have contributed to the development of the model's present structure and data. In addition to those already mentioned, I would particularly like to recognize the contributions of Herman Attinger, Bob Bain, Bill Balson, Richard Barendson, John Bell, Horace Brock, Bill Clark, Lou Deziel, Jack Eddington, Bob Fullen, John Hackworth, Arvind Jain, Ellen Leaf, Marley Mandt, Donna Oman, and Steve Regulinski.

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I. INTRODUCTION

The SRI-Gulf Energy Model is a highly-detailed regional and dynamic model of the supply and demand for energy in the United States. The model was originally developed in 1973 to analyze synthetic fuels strategy for the Gulf Oil Corporation and has subsequently been extended and widely used in energy analyses sponsored by several agencies of the U.S. Government, the Electric Power Research Institute, and a number of private sector organizations. Using the same basic methodology, a world energy model, a new computer modeling language, and applications to multi-party decisions are currently under development. Several publications have described the detailed results and applications of the U.S. version of the model and summarized the basic concepts of the generalized equilibrium modeling methodology underlying the model [1, 2, 3, 4, 5, 6]. This report, however, is the first comprehensive description of the generalized equilibrium modeling methodology.

The generalized equilibrium modeling methodology represents a synthesis of many modeling techniques and has evolved from more than ten years of research by the author and his colleagues. The initial work was stimulated by the limitations of both mathematical programming and simulation and resulted in the development of new methodology for the coordinated decomposition of complex decision or optimization problems involving many resources, time, uncertainty, and multi-attribute preferences [7, 8, 9, 10]. The research described here extends the methodology to decentralized or market decision problems where there is no single overall objective function. In its new form, the methodology resembles many aspects of general economic equilibrium theory, and for this reason, and also to emphasize the differences, the term ***generalized equilibrium modeling*** best describes the particular synthesis of basic modeling techniques used in the SRI-Gulf model.

This report begins with an introduction to the conceptual framework of generalized equilibrium modeling that emphasizes (1) the need to focus modeling efforts on decisions and (2) the coordinated decomposition of complex decision problems using iterative methods. This conceptual framework is followed by a description of the structure of the current SRI-Gulf model and a detailed development of the process relations that comprise the model. Finally, the network iteration algorithm used to compute a solution to the model is described and the overall methodology is compared with other modeling methodologies.

II. A CONCEPTUAL FRAMEWORK FOR GENERALIZED EQUILIBRIUM MODELING

The conceptual framework for generalized equilibrium modeling is based on the twin foundations of decision analysis and coordinated decomposition. Implementation of the conceptual framework is in terms of a network of process models and an iterative solution algorithm.

Need to Focus on Decisions

The need to focus modeling efforts on specific decision problems is best illustrated by considering what happens when the focus of a model is not defined. Every modeler must make decisions on the appropriate level of detail and information to include in a model. If there is no way to judge whether additional detail in the model or additional information-gathering is worth the cost, the model either tends to be unnecessarily large or arbitrary decisions must be made to leave important detail and information out of the model. On the other hand, if a decision or a class of decisions is identified as the focus of a modeling effort, then sensitivity analysis that shows the effect of additional model detail and information on the choices among decision alternatives can be used to decide what detail and information are most important. A model that provides insight into the decision problem is the result. Many modelers resist focusing on decisions with the result that the end product of their efforts is generally their model rather than insight communicated to decision-makers.

Coordinated Decomposition of Complex Decision Problems

Most decision problems of importance are complex and therefore it is useful to simplify models for such problems by employing the principle of "divide and conquer' or decomposition. The idea of decomposition is to break a complex model of a decision problem into a number of simpler submodels and then to model the original decision problem by the coordinated use of the submodels. The value of coordinated decomposition in modeling is that it reduces the cost of complex models and provides direct insight into the problem as result of the structure of the model.

Decomposition of a model usually begins by distinguishing between the *exogenous* decisions that must be made immediately by the decision-makers who will use the insight from the model and the *endogenous* decisions that are either future decisions by the same decision-makers or immediate and future decisions by other decision-makers that bear on the exogenous decisions. For example, if the exogenous decision were whether a particular company should build a synthetic fuels facility, it would be necessary to define and model the endogenous decision processes of other companies and the government, as they affect the price received for synthetic fuels and the cost of production.

The analysis of the exogenous decision problem is related to the model of the endogenous decision processes and physical systems in a way that is illustrated by Figure 1. The exogenous decision alternatives are described at the bottom of the figure. On the left, the information available to the exogenous decision-makers is described in terms of probability distributions on the exogenous state variables. Together, the exogenous state and decision variables interact within the model of the endogenous decision processes to determine, on the right, the exogenous outcome variables that are important to the decision-makers. Using a model of the exogenous decision-makers' preferences, the relative value of each alternative to the exogenous decision-makers can be determined. These relative values of alternatives, along with other insights, are then communicated to the decision-makers as a basis for their decisions.†

The endogenous model in the center of Figure 1 must describe the world beyond the decision-makers. More specifically, this model must describe how the exogenous state and decision variables affect the exogenous outcome variables. Since it is never practical to model the world beyond the decision-maker at the level of detail necessary to evaluate a wide range of exogenous decision problems, the endogenous model must emphasize those aspects of the world that are crucial to the exogenous decision problem while modeling the rest of the world in much less detail. The model of the rest of the world cannot be ignored, however, since assumptions regarding less important aspects of the model must at least be made explicit in order to test the model.

The original SRI-Gulf model, for example, was developed to analyze synthetic fuels policy for Gulf Oil Corporation. The executives of Gulf Oil were the exogenous decision-makers; the exogenous decision alternatives represented investments in several synthetic fuels technologies, the relative value of alternatives was measured in terms of company discounted profits; and, the endogenous model described in great detail the alternative synthetic fuels technologies and all other major energy

† This report assumes the reader is generally familiar with the methods of decision analysis as described in [12].

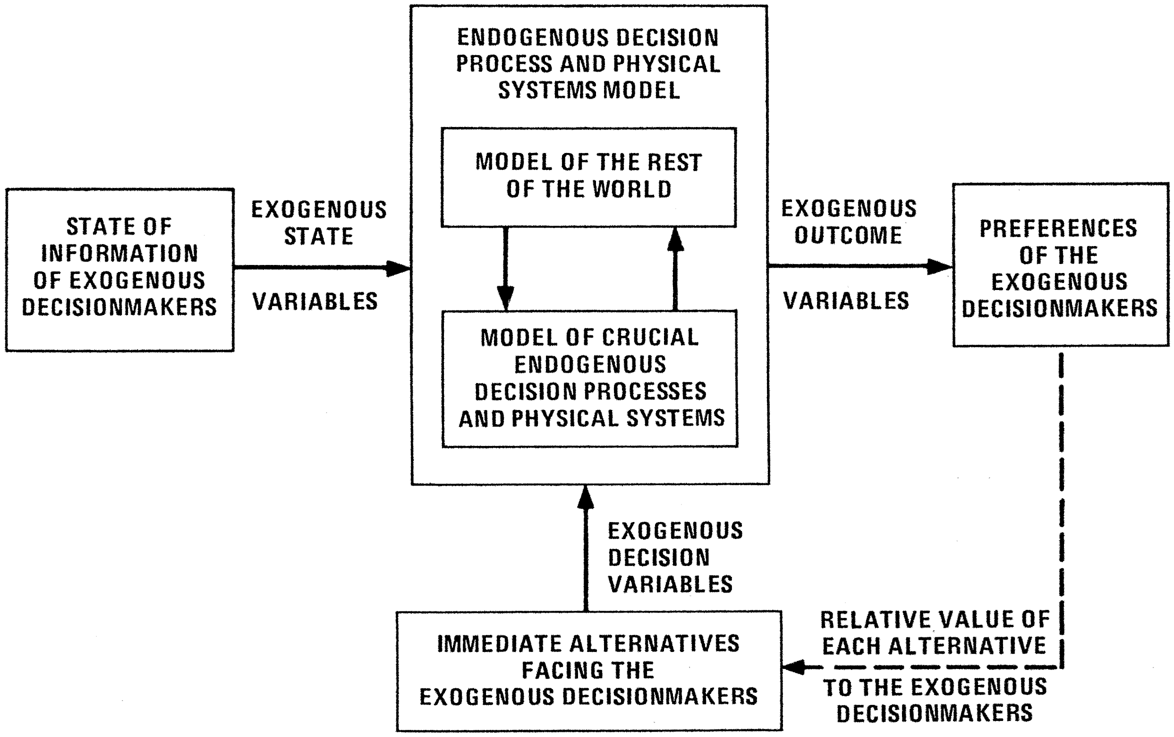

FIGURE 1. CONCEPTUAL STRUCTURE OF A GENERALIZED EQUILIBRIUM MODEL

technologies and the important U.S. energy market decision processes as they affected the evaluation of Gulf's decisions. In this case, the model of the rest of the U.S. economy and energy supply and demand in foreign countries was much less detailed. In subsequent applications of the model, detail has been eliminated or enhanced as needed for specific exogenous decision problems.

Often the model of the crucial endogenous decision processes and physical systems is still complex, thus requiring further decomposition. At this level, decomposition can be pursued by defining the basic decision and physical processes that must be included. If the decomposition is pursued to the appropriate level, then the submodels can be constructed from a few relatively simple process models that are interconnected using a *network* and an *algorithm* as described below.

Basic Elements of a Generalized Equilibrium Model

Once a decision problem and a model have been conceptualized as a decomposed set of submodels, the overall model can be implemented using the following three basic elements of a generalized equilibrium model: (1) *processes* describing the fundamental submodels; (2) a *network* describing the interactions among the processes; and (3) an *algorithm* for determining the numerical values of all of the variables in the model.

A process is a model of a subsystem or sector of the overall system being modeled that is identified in decomposing the overall model. A higher level process may also be constructed using a subnetwork of processes. Depending on the level of required detail, a process may represent an entire national economic or natural system or could represent a fundamental technology or decision-making process.

A process is characterized by a set of mathematical relations or equations that are usually formalized as computer code. These relations may be economic (derived statistically from historical data) or subjective (derived from direct expert judgement) or a combination of the two. In each process, two types of process relations can generally be defined. The first is *physical* and describes how physical flows interact over time. The second type of process relation is *behavioral* and describes human choices and the information these humans have about other processes and future physical flows and behavioral choices. The next section of this report describes in detail the physical and behavioral relations for the basic processes used in the SRI–Gulf model.

A network defines the links among the processes. Figure 2 is a simplified, schematic diagram that illustrates the network for the SRI–Gulf model. In this model,

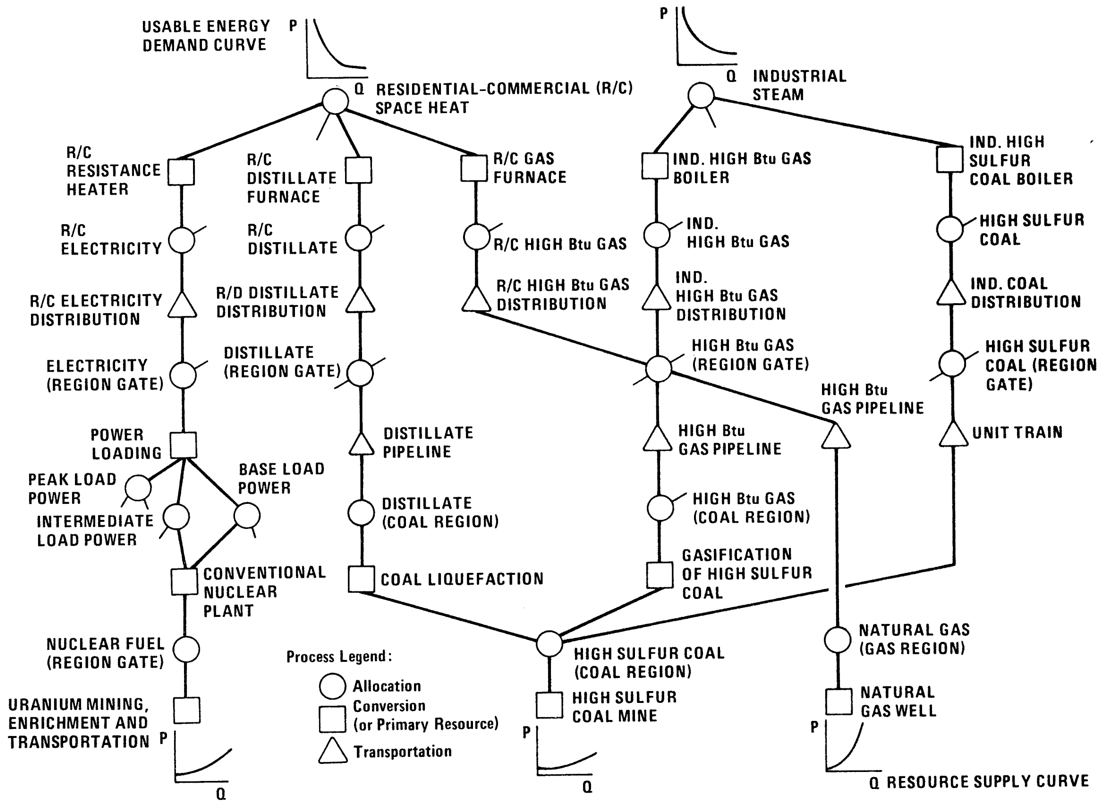

FIGURE 2. SCHEMATIC DIAGRAM OF THE NETWORK STRUCTURE OF THE SRI-GULF MODEL

the links are expressed as prices and quantities of energy products. Other links in the model that are not shown in Figure 2 can express environmental controls and outcomes, the relationship of the energy sector to the economy, and constraints on prices or quantities. At the top of this network are processes describing the end-use demands for energy, and at the bottom are processes describing primary resource supply. In between is a network of other processes describing market behavior, conversion and transportation in the entire energy system. The actual SRI-Gulf model network currently has about 2,700 processes of the types illustrated in Figure 2. The structure of this network will be described in more detail later in this report.

The algorithm used to solve a generalized equilibrium model must find the set of variables (usually prices and quantities) that satisfy the physical and behavioral relations embodied in the processes and the linkages among the variables as defined by the network. Generally, explicit solution of the model is not possible and a solution for a given set of parameters must be determined using iterative techniques that successively adjust the prices and quantities until a solution is found. In the next section, the particular algorithm used in the SRI-Gulf model will be described. At this point, it is important to note that the algorithm used to solve a model should take advantage of the natural structure of a problem rather than imposing arbitrary restrictions on the problem structure in order to guarantee that the algorithm will work.

III. OVERVIEW OF THE SRI-GULF MODEL

The SRI-Gulf energy model has been used for analysis of a wide range of Corporate and government energy decision problems. In each case, the model was modified to enhance or eliminate detail as required to evaluate the exogenous decision alternatives of each problem. Since the detailed structure of the model is constantly changing, this report will simply illustrate the model's current structure as a basis for describing the methodology. The reader who is interested in detailed structural assumptions is referred to other publications [3, 4].

The specific structure of the current SRI-Gulf model reflects, as it should, the decision problems for which it has recently been used. Specifically, the model has been used to analyze government and private industry policy on synthetic fuels and commercialization and prioritization of research and development on energy supply technologies. As a result, the present model contains great detail on these technologies and other competing techhologies. Also, detail on energy resource supplies, energy markets, energy transportation and end-use energy conversion is incorporated because these aspects of the energy system determine the market for new energy supply technologies. A different emphasis on detail would be required for other energy decisions such as those concerning regulation of energy prices, energy conservation, and environmental control.

Model Structure

The structure of the SRI-Gulf model is best illustrated by the schematic network diagram shown in Figure 2 and the map and regional and transportation structure shown in Figure 3.

In the current model network, 17 end-use demands are modeled for each of the 9 demand (U.S. census) regions. An example of the detail in the demand sectors of the model is shown in Figure 4 for the residential and commercial sectors. Note that the end-use demand for space heat is in terms of the energy output of a furnace within a residence and fuel demands for space heat are derived as the inputs to the furnace. This is in contrast to most energy models that only treat fuel demands and do not explicitly consider end-use energy demands. Moreover, the technologies and energy products shown in Figure 4 cover a wide range of present and future technologies and products. Comparable detail is included in the industrial and transportation demand sectors.

Conversion of energy from natural to synthetic products occurs in the demand regions and in some of the supply regions denoted in the map in Figure 3. The demand region conversion process detail for producing hydrogen, electricity, and low Btu gas is shown in Figure 5. Most of the detail in that figure relates to electricity generation for base, intermediate, and peak load power demands. Synthetic high Btu (pipeline) gas, shale, and coal synthetic crude oils, methanol, and other synthetic products are produced in the shale and coal supply regions.

Interregional transportation is described by a transportation network for high Btu gas (both natural and synthetic). As Figure 3 shows, only the major current or potential future transportation routes are included. Comparable transportation networks for all other products and resources are specified in the model.

The supply of primary resources such as coal, nuclear fuels, shale, geothermal steam, crude oil, and natural gas is described by processes defined for each appropriate supply region. An important and unique aspect of those supply process models is their use of long-run supply curves such as those shown for crude oil in Figure 6. Within the resource process models, the prices of primary resources are given by the marginal cost from these curves plus an economic rent term that represents resource scarcity. The computation of economic rent is described in the next section of this report.

Not shown in the network diagram of Figure 3 is the simplified model of the economy and population growth used to derive the end-use demands. This end-use demand model takes projections of aggregate economic activity and population growth and determines sector activity and the price elasticity of the demand for end-use energy in each sector. Also not shown in Figure 3 are the factors of production or secondary materials consumed by processes. The model incorporates simplified secondary material production processes to describe the effect of increases in the demand for secondary materials on the prices of secondary materials and hence on the prices of products.

The dynamic structure of the model spans the next fifty years. Within each of the process models, installation and retirement of facilities over time is modeled, and in the case of primary resources, depletion over time is modeled. Of central importance to modeling of capacity expansion decisions are the dynamic aspects of forecasting (the formation of expectations about future variables) and the modeling of time delays or lags introduced through the normal functioning of the decision processes. For example, the model uses estimates

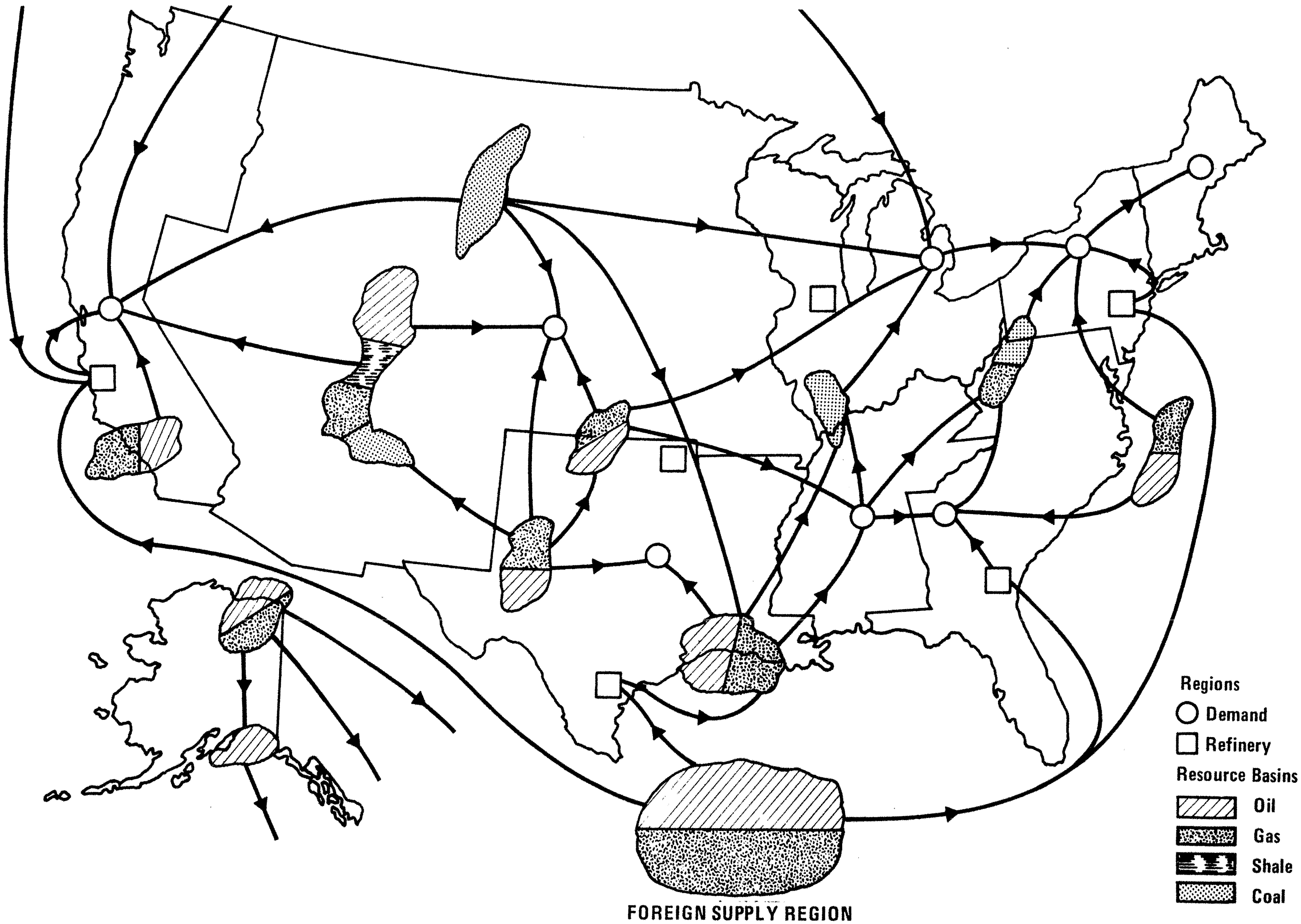

FIGURE 3. REGIONAL AND TRANSPORTATION NETWORK STRUCTURE OF THE SRI-GULF MODEL

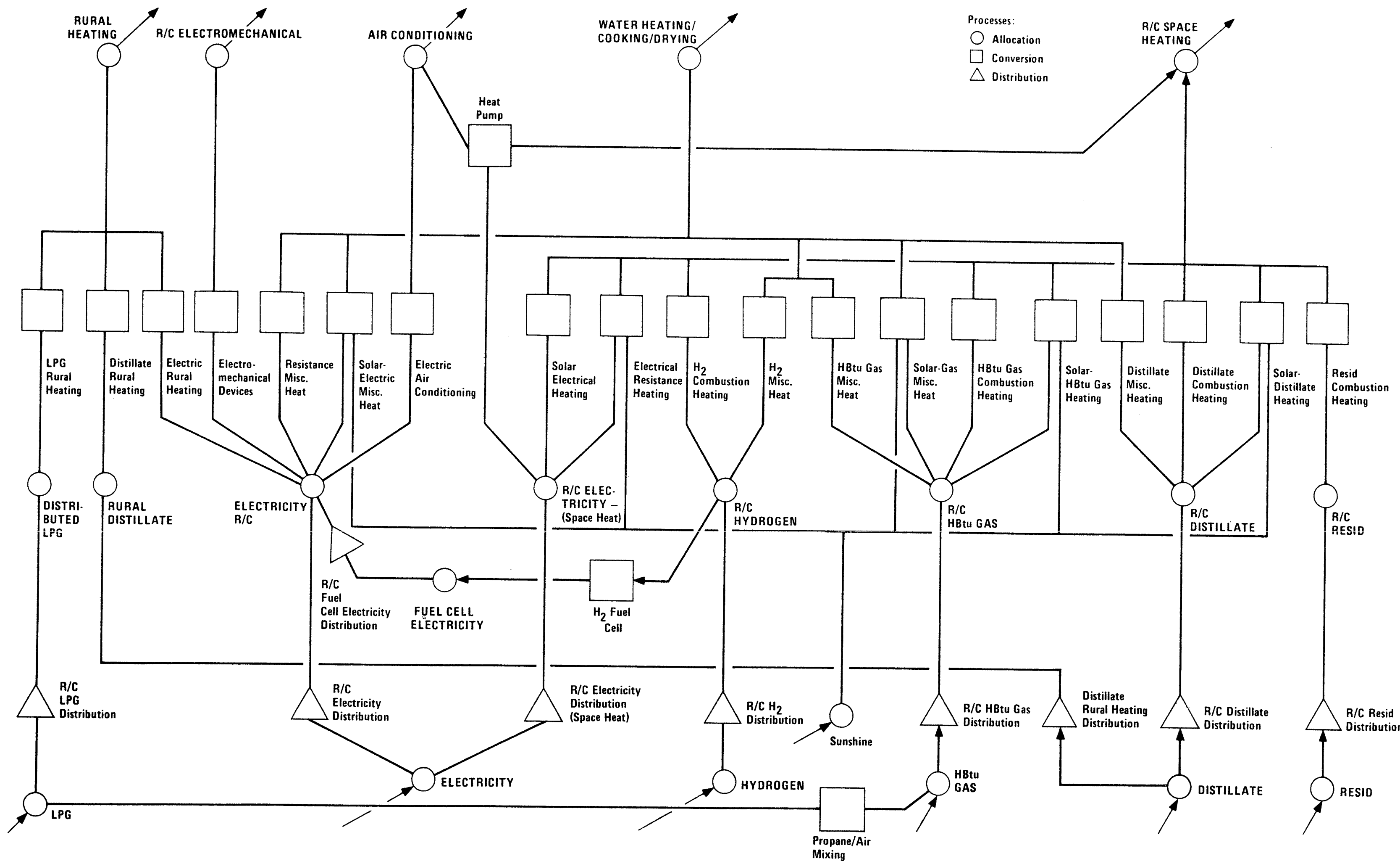

FIGURE 4. RESIDENTIAL/COMMERCIAL SECTOR DETAIL

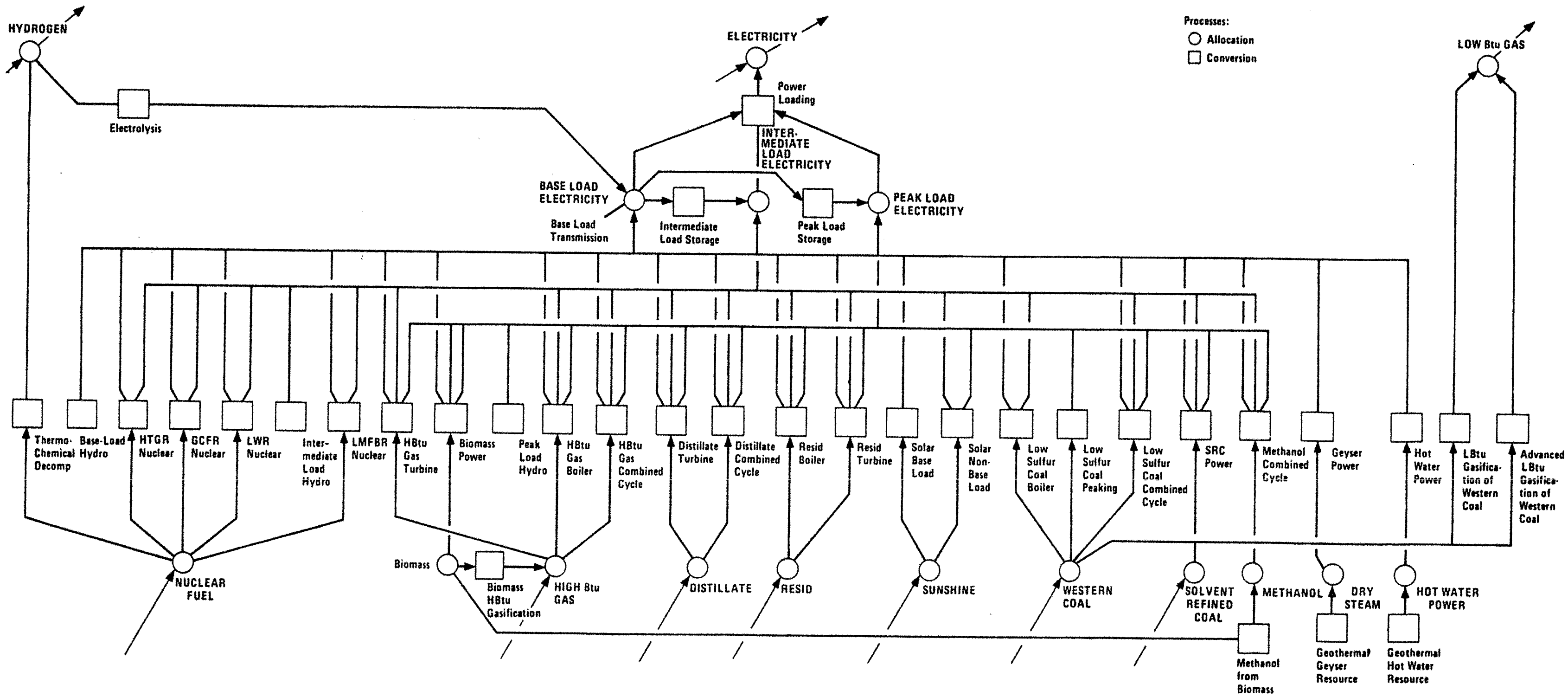

FIGURE 5. DEMAND REGION CONVERSION SECTOR DETAIL

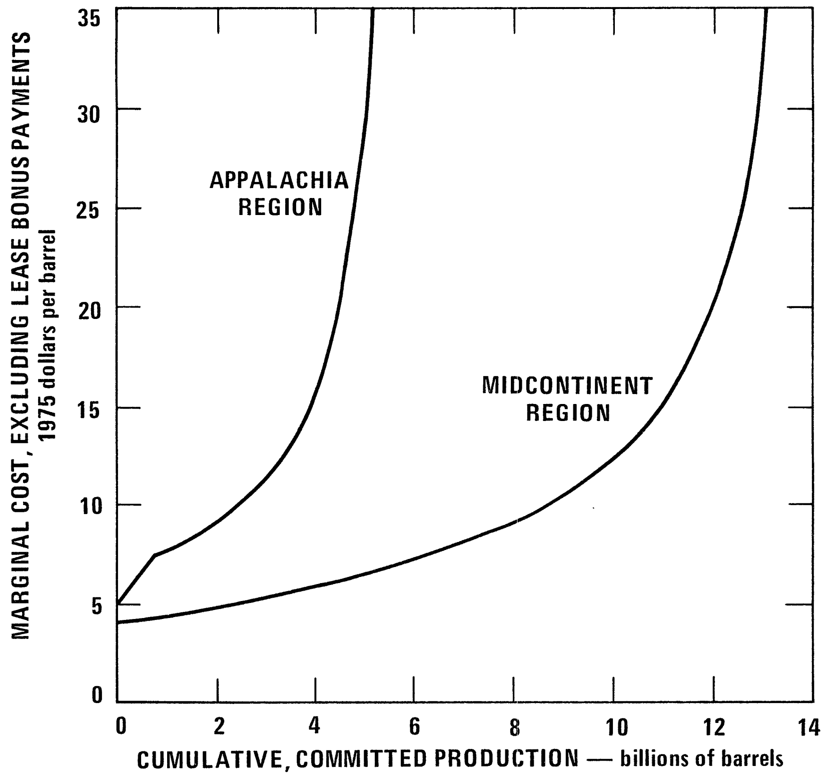

FIGURE 6. LONG-RUN SUPPLY CURVES FOR CRUDE OIL

of future prices of primary resources to determine economic rent on resources in earlier time periods. These and other dynamic aspects of the model are described in detail in this report.

This brief description of the structure of the SRI-Gulf model should convey the high degree of decomposition of the U.S. energy market into processes which describe the technology and decision behavior of each sector of the energy market. Because of the high level of detail in the model structure, the structure within each of the process models can be relatively simple and thus easily characterized by straightforward process models. Most importantly, the assessment of model parameters can be done at the level of detail appropriate to the expertise and data that is available. Thus specialists in each element of the energy market can provide detailed inputs and review of the model in terms that they normally use. In the next chapter, the structure of these process models is described in detail. First, however, it is necessary to describe how the prices and quantities of energy materials, and the implicit endogenous decisions on technology selection that form a solution to the model, are determined.

Network Iteration Algorithm

The basic idea of the network iteration algorithm is to start with initial rough estimates of prices and quantities for all the periods and for all energy products produced or transported by the processes and then successively adjust these prices and quantities until all of the relations embedded in the process models are satisfied. The resulting set of prices and quantities is usually called the *equilibrium solution* since it is the solution that balances or satisfies all of the relations embedded in the processes. The equilibrium solution, however, will reflect whatever market imperfections and human behavior are built into the processes including shortages introduced by price controls and other restrictions; idealized, perfect market behavior need not be assumed in a generalized equilibrium model. Moreover, the equilibrium solution is dynamic and the solution for a given time period depends on solutions in past and future time periods.

The algorithm in the SRI-Gulf model takes advantage of the network structure of the model in iterating up and down the network (see Figure 2) computing tentative prices on the upward iteration and quantities on the downward iteration. The upward pass on a given iteration begins with estimates of primary resource prices and computes product prices and end-use energy costs based on a subset of the relations in each process. On the downward iteration quantities are computed using the remaining relations in each process.

In iterating up and down the network, the prices or quantities of each process output or input are computed for all time periods before moving to the next output or input. An alternative iterative scheme would be to iterate back and forth through time and within each single time period move up and down the network only once. For the present model, the first scheme is less expensive from a programming and computation point of view. The astounding feature of these iterative schemes is that they cost very little more to program and compute than algorithms that impose severe restrictions on the dynamic structure of the model. More discussion of this point is contained in Chapter IV.

Having described the generalized equilibrium modeling concepts and the overall structure of the SRI-Gulf model, the next section describes more specifically the details of the relations embedded in each of the processes in the model.

IV. BASIC PROCESSES IN THE SRI-GULF MODEL

The SRI-Gulf model decomposes the U.S. energy market into thousands of process models; each process is different in terms of specific parameter assumptions, but each can be implemented as one of a very few basic processes. Each of these basic processes is relatively simple and therefore easy to understand and implement. However, when the basic processes are linked together within the network and by the iterative algorithm, it is possible to model complex market phenomena that would be difficult to model except through a decomposition approach.

The basic processes of the current (1976) SRI-Gulf model are described in this section in great detail. Emphasis is placed on precisely describing the mathematical relations in each of the basic processes in order to illustrate and document how the SRI-Gulf model is implemented. These relations, however, are not fixed and can easily be changed if they are not adequate for the decision problem at hand. The basic processes in the current model are as follows:

1. *Simple Conversion Processes* describe the technology and economics of converting one energy material to another (i.e., synthetic gas from goal and residential space heat from gas).

2. *Allocation Processes* describe the allocation of demand among competing sources of supply (i.e., allocation of gas demand among alternative sources and space heat demand among alternative furnaces and fuels).

3. *Primary Resource Processes* describe the depletion and pricing of energy resources (i.e., natural gas and coal).

4. *End-Use Demand Processes* describe the growth in demand for usable energy over time and the effect on demand of changes in prices (i.e., space heat and industrial steam).

5. *Transportation Processes* describe technology and economics of moving an energy material from one location to another (i.e., natural gas pipelines and coal unit trains).

6. *Complex Conversion Processes* (i.e., refineries and electric power generation).

7. *Secondary Industry Processes* describe the impact on secondary material prices of changes in demand for secondary materials used in construction of new energy facilities (i.e., pressure vessels and surface mining equipment).

The structure of each of the basic processes is similar. Each basic process has two types of relations: *physical relations* and *behavioral relations*. Physical relations describe the flow of materials within a process, whereas behavioral relations describe the decision-making behavior that sets prices and quantities. As will be explained, physical relations are naturally computed or solved for each process stepping forward through time and behavioral relations are computed for each process stepping backward in time (analogous to the roll-back operation of decision trees and dynamic programming). Also, the physical relations are used on the downward iteration through the network, whereas the behavioral relations are used on the upward iteration. Other orderings of iteration steps and specification of the relations are possible, and may be appropriate in certain applications.

Overview of Notation

In describing the process relations, it is useful to simplify the notation as much as possible since a precise notation would require so many subscripts on each term as to be unreadable.

In this report, the time dimension will be defined in terms of discrete, one-year-long periods. The model uses three-year time periods, and is programmed for variable time periods but translation of the relations from one-year to three-year periods or to variable time periods is straightforward. Time will be indexed by the variable t. For example, the quantity (flow) and price of a particular material or process input or output will be denoted for a given year by the notation q(t) and p(t) respectively. The variable t will run from 0 to T, the horizon of the model. Unless otherwise stated, all financial flows will be expressed in current (inflated) currency.

Subscripts on variables are not used wherever implicit subscripts are sufficient. For example, where several inputs and outputs are involved, a subscript is necessary either to distinguish between inputs and outputs of processes, or among inputs and outputs. For example, in the case of a simple process, $q_f(t)$ denotes the input (feedstock) quantity, and q(t) denotes the output quantity.

For each process, a simple diagram is included to provide easy reference to the notation and to avoid lengthy definitions of process parameters and other variables. The definitions of the variables in this report are descriptive, but will not usually be sufficient for practical work. More detailed definitions are given in other publications.

Simple Conversion Process

A simple conversion process is used in the model to describe representative energy technologies. These technologies represent a particular conversion industry such as coal gasification. Depending on the degree of detail required, several technologies or simple processes may be used to describe a given industry. The choice among technologies is made within the allocation processes using prices provided by the simple comparison processes. These prices are the inputs to the allocation process for a sector. Because the allocation process treats the choice of technology decisions, the simple conversion process can concentrate on representing the economics of a well-defined technology.

Figure 7 summarizes the inputs, outputs, and basic data required by a simple conversion process and gives brief definitions of the variables. Most of the parameters of the model are expressed as general functions of time to allow for technological change, differential inflation and general inflation.

The physical relations for a simple conversion process are straightforward physical accounting flows and will be described first. The notation used is either self-explanatory from the names of each relation, or from the labels in Figure 7.

PHYSICAL RELATIONS:

(1) Material Flow

$$q_f(t) = q(t)\,\frac{1}{e}$$

(2) Second Material Quantity

$$w_j(t) = c_a(t) \cdot k(t) \cdot s_j$$

(3) Capacity Additions

$$c_a(t) = c(t) - c(t-1) + c_r(t)$$

(4) Capacity

$$c(t) = \max\,[q(t),\, c(t-1) - c_r(t)]$$

(5) Capacity Replacements

$$c_r(t) = c_a(t-L)$$

(6) Initial Conditions

$$c(0) = q(0)\ ;\ c_r(t) = \frac{1}{2L} \quad t \leq L$$

As can be seen, these physical relations are conveniently solved in forward time. In the actual model, the process thermal efficiency is expressed as a function of time to represent technological change. This generality is not shown here only because of the notational complexity that arises in the description of the behavioral relations when thermal efficiency is not fixed over time.

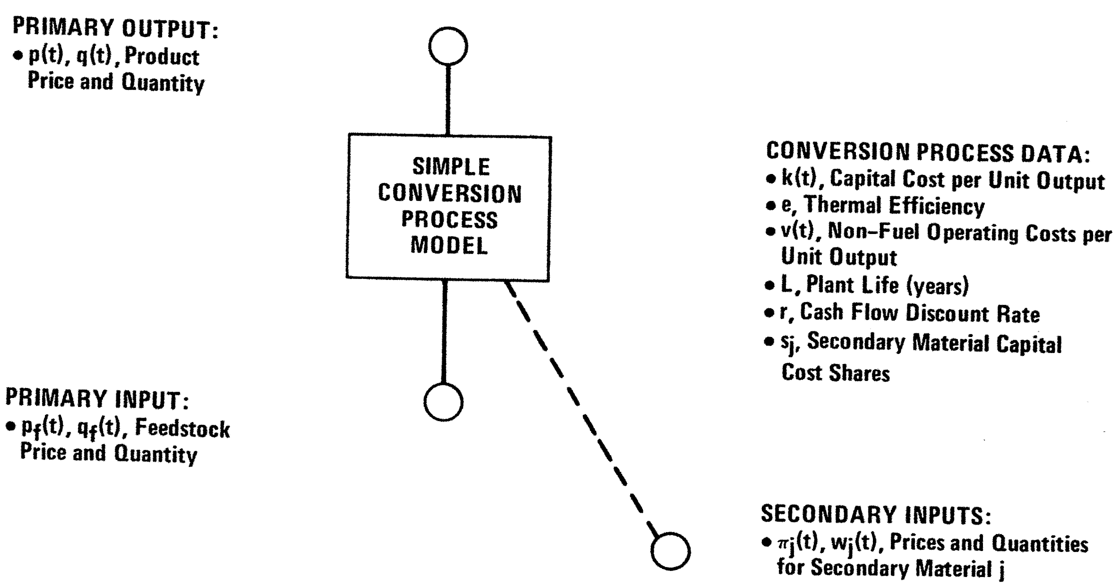

FIGURE 7. SIMPLE CONVERSION PROCESS NOTATION

that arises in the description of the behavioral relations when thermal efficiency is not fixed over time.

BEHAVIORAL RELATIONS:

The criteria for investment and pricing in a privately-owned energy facility are typically given by the net present value or discounted cash flows resulting from investment and operation of the facility. For a marginal new unit of capacity, the net present value is as follows:

(7) Net Present Value

$$NPV(t) = -k_d(t) + (1-f_{max}) \sum_{\tau=t}^{t+L} \frac{p(\tau) - \phi(\tau)}{(1+r)^{\tau-t}}$$

$$\begin{pmatrix} \text{Net Present} \\ \text{Value to Owner} \\ \text{After Taxes} \end{pmatrix} = \begin{pmatrix} \text{Present Value} \\ \text{of Capital Cost} \\ \text{After Taxes} \end{pmatrix} + \begin{pmatrix} 1- \begin{matrix}\text{Tax}\\ \text{Rate}\end{matrix} \end{pmatrix} \begin{pmatrix} \text{Present Value of} \\ \text{Product Price Less} \\ \text{Cost Summed Over} \\ \text{Years of Operation} \end{pmatrix}$$

In (7), the following relation is used:

(8) Operating Cost

$$\phi(t) = v(t) + p_f(t) \frac{1}{e}$$

In an unregulated market, it is a frequent observation that prices tend to be higher in a rapidly growing market than in a slowly growing or declining market. In a slowly growing market, the net present value of the marginal unit of capacity would tend towards zero at the industry rate of return (discount rate) r. Under this condition, where

$$NPV(t) = 0,$$

we can solve relation (7) for the required price of a plant's product in its first year of operation, which results in the following relation:

(9) Product Price Under Slow Growth

$$p(t) = \phi(t) + \left[\frac{k_d(t)}{1-f_{tax}} - \sum_{\tau=t+1}^{t+L} \frac{p(\tau) - \phi(\tau)}{(1+r)^{\tau-t}} \right]$$

The term in the brackets of (9) can be viewed as a capital charge, which when added to the first year operating cost gives the required first year product price. Note that the capital charge reflects the difference between the tax-adjusted discounted capital cost of the facility and the value of the facility in future years. Moreover, relation (9) must be solved backward in time, beginning from a set of terminal value assumptions about prices beyond the horizon of the model. The terminal value model uses a simple infinite horizon capacity replacement model to set future prices in the same way that (9) can determine the required prices up to the model horizon.

In a period of rapid growth of an industry, prices in capital-intensive industries tend to increase more than for industries where the capital investment is smaller and capacity expansion is easier. Relation (10) adjusts the product price in proportion to the capital charge rate under slow growth and as a function I(t) of the growth rate R(t).

(10) Product Price Under Rapid Growth

$$p(t) = \phi(t) + \left[\frac{k_d(t)}{1-f_{tax}} - \sum_{\tau=t+1}^{t+L} \frac{p(\tau) - \phi(\tau)}{(1+r)^{\tau-t}} \right] I(t)$$

where

(11) Growth Multiplier

$$I(t) = 1 + R(t)\,\rho_g \qquad R(t)>0$$

ρ_g is a single parameter describing the sensitivity of price to growth rate, and

(12) Growth Rate

$$R(t) = \frac{c_a(t)}{c(t)} \qquad c_a(t)>0$$

In a period of excess capacity in an industry, prices tend to be lower, but are unlikely to fall below the variable operating cost of the plant; otherwise, the plant would be shut down. Relation (10) also applies in this case, but the growth multiplier is replaced by the excess capacity multiplier.

(13) Excess Capacity Multiplier

$$I(t) = \max(0, 1-E(t)\rho_d) \qquad E(t)\geqslant 0$$

where ρ_d is a single parameter describing the sensitivity of price to excess capacity, and

(14) Excess Capacity Fraction

$$E(t) = \frac{c(t) - q(t)}{c(t)} \qquad c_a(t)=0$$

The resulting relation between price and growth rate and excess capacity is summarized in Figure 8.

Among the details of the simple conversion process not described above are the model of technological change and the adjustments to the present value of

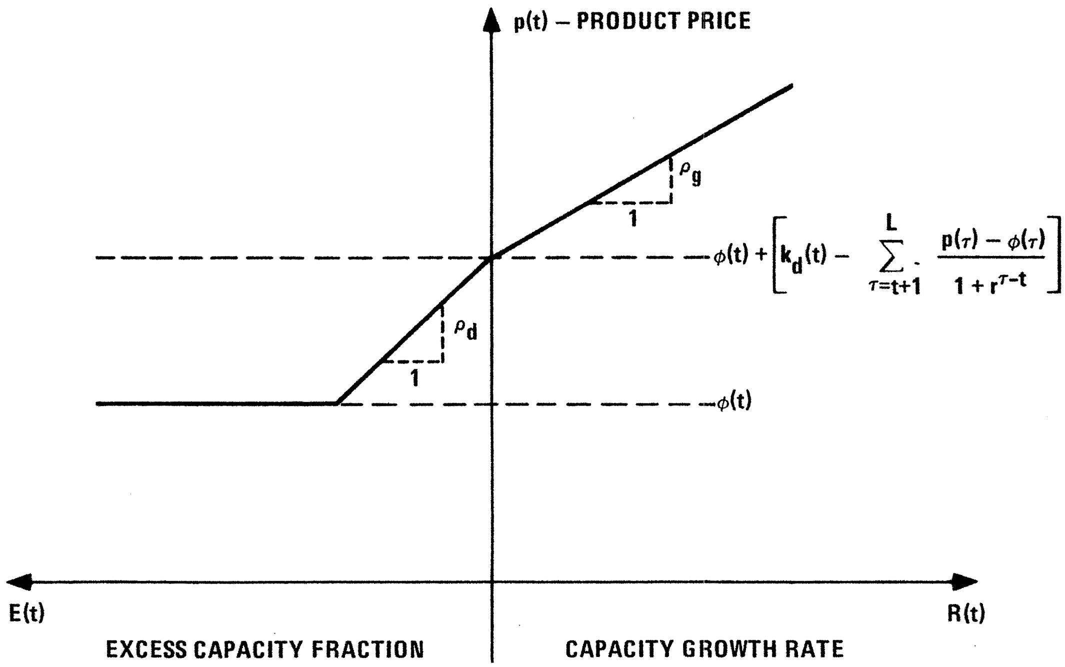

FIGURE 8. EFFECT OF CAPACITY GROWTH RATE AND EXCESS CAPACITY ON PRODUCT PRICE

capital cost of the facility for secondary materials prices, taxes, depreciation, and interest during construction.

Technological change on capital is described by the learning curve illustrated in Figure 9. A capital cost premium is incurred for use of a technology before the date of commercial availability, t_a, and evolutionary technological change occurs after commercial availability. The formula used to represent technological change is as follows:

(15) Technological Change on Capital Cost*

$$k(t) = k_o \begin{bmatrix} 1+\rho_e(t_a-t) & t<t_a \\ f_\infty + (1-f_\infty)\,(1+\rho_c)^{t-t_a} & t\geq t_a \end{bmatrix}$$

where ρ_e is the technological change rate prior to commercial availability and ρ_c is the technological change parameter after commercial availability.

General inflation on capital cost is accounted for by the following adjustment:

(16) Inflation Adjustment**

$$k_i(t) = k(t)\,(1+r_{inf})^t$$

*A similar formula is used to model technological change in process thermal efficiency.

**A similar formula is used to adjust operating cost for inflation. Any difference in inflation rate for capital and for operating costs can be reflected in technological change.

The capital cost determined by (16) must also be adjusted for changes in secondary material prices according to the fraction of capital cost allocated to each secondary material under nominal secondary material prices. This is accomplished as follows:

(17) Secondary Materials Cost Adjustment

$$k_a(t) = \sum_{\text{all } j} k_i(t)\, s_j \pi_j(t)$$

Calculation of the effects of taxes, depreciation and interest during construction on the present value of capital cost used in (7), (9), and (10) is typically very complicated. The calculation, however, is simplified by the following formula:

(18) Present Value of Capital Cost

$$k_d(t) = \underbrace{k_a(t)\cdot(1-f_I)}_{\substack{\text{investment}\\ \text{tax credit}\\ \text{effect}}} \underbrace{\left[1 - \frac{\frac{2}{t_L} f_{tax}}{1 - \frac{1-\frac{2}{t_L}}{1+r}}\right]}_{\substack{\text{tax credit due}\\ \text{to depreciation}}} \underbrace{\left(\frac{1+r}{1+r_{inf}}\right)^{\frac{t_c}{3}}}_{\substack{\text{interest and}\\ \text{inflation during}\\ \text{construction}}}$$

This formula assumes double declining balance accounting for tax purposes over a tax life of t_L years and

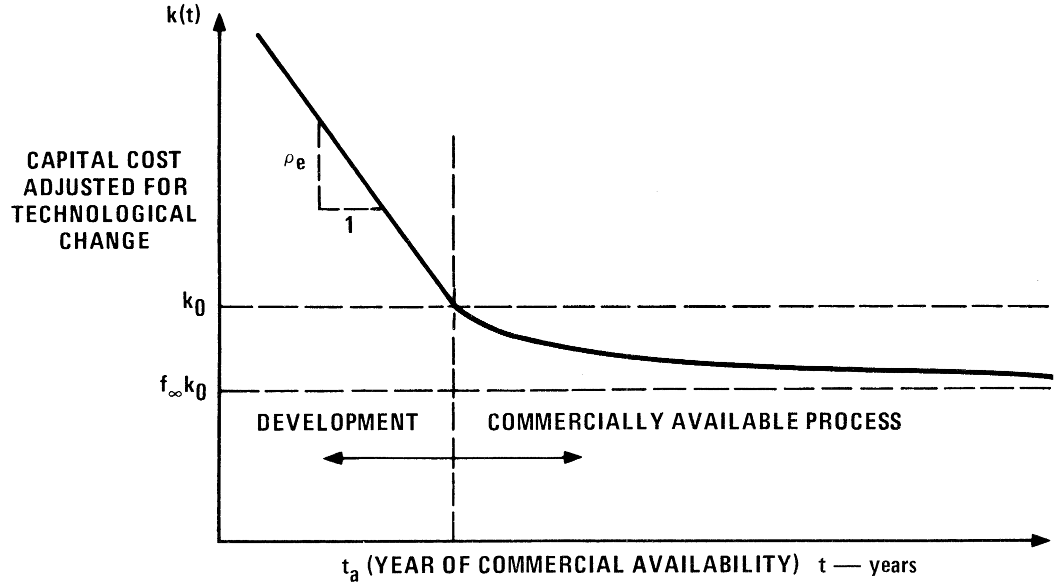

FIGURE 9. TECHNOLOGICAL CHANGE FUNCTION

financing during construction and operation at the discount rate, r. Capital spending and inflation during construction is treated as if it were a single expenditure at the two-thirds point of the construction period of length t_C. The investment tax credit at the rate f_I is assumed to be taken at the time of capital expenditure.

This concludes the description of the simple conversion process relations. While the simple conversion process is completely described at this point, it is not possible to understand fully the behavior of the simple conversion process model in isolation from other processes. The simple conversion process describes the economics of a representative technology within the industry. The allocation process which is described next acts in combination with the simple conversion processes to model the economics of a sector of the energy economy. In essence, the allocation model extends the representative technology modeled by the simple conversion process to a sector model comprising a distribution of technological alternatives and decision-making processes.

The simple conversion process and the allocation process also interact dynamically. The prices computed in a simple process assume knowledge of future prices as is evident in relation (9). The allocation process recognizes the high degree of foresight built into the prices and adjusts the response accordingly. Linked together, the simple conversion process and the allocation process describe markets characterized by imperfect foresight.

Allocation Process

An allocation process in the model typically represents a market decision-making process where buyers and sellers trade at a price. Allocation processes where the decisions are regulated by government action are described later. In modeling an allocation process, perfect market assumptions of cost minimizing behavior should be allowed but not required. The prices that are input to an allocation process are prices of products from representative conversion processes. An allocation process that responded sharply to small differences in prices from representative technologies (as would be the case if the demand were allocated entirely to the lowest price source) would overstate the market response to prices. Thus, the allocation process described here produces allocations that respond continuously to changes in representative product prices. This continuous response to prices allows the overall model to reflect the range of technologies, costs, and preferences that occur in actual markets.

The physical relations for the allocation process insure a material balance within the process and also segment total demand into existing demand and new demand. This segmentation of demand into existing and new categories is used in the modeling of behavioral lag. The behavioral equations describe the static allocation of demand in terms of simple market share curve and a simple market penetration (behavioral lag) curve that reflects lags or time delays in responding to price changes.

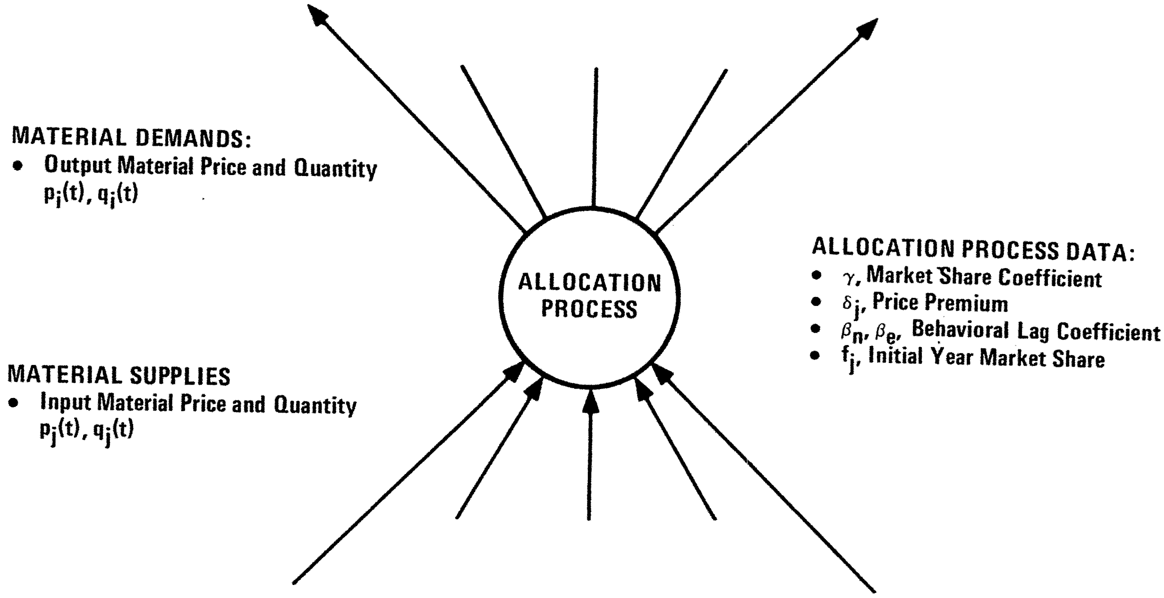

FIGURE 10. ALLOCATION PROCESS NOTATION

PHYSICAL RELATIONS:

(19) Physical Flow

$$\sum_{\text{all } i} q_i(t) = \sum_{\text{all } j} q_j(t)$$

or "total input" = "total output"

(20) Total Quantity

$$q(t) = \sum_{\text{all } j} q_j(t)$$

(21) Existing Quantity Demanded

$$q^e(t) = \min\,(q(t),\ q(t-1))$$

(22) New Quantity Demanded

$$q^n(t) = q(t) - q^e(t)$$

(23) Initial Market Share Conditions

$$q_j(0) = f_j q(0)$$

where f_j is the initial market share allocated to source j at the initial period. These initial market shares, together with initial end-use demand estimates and conversion and transportation process efficiencies, determine all of the quantities for the initial period of the model.

BEHAVIORAL RELATIONS:

The behavioral relations of the market share model combine a static and dynamic component. The static allocation of demand $q_j(t)$ to a given source j is inversely related to the price of the j^{th} source according to the following relations:

(24) Market Share

$$\hat{q}_j(t) \propto \left(\frac{1}{p_j(t)}\right)^{\gamma}$$

In cases where price is an insufficient criterion for allocation, provision is made for price premiums to be added to the market prices to reflect inconvenience, aesthetic differences, or other differences among sources that are not reflected in the price. In the presence of price premiums the allocation to source j is inversely related to the price plus premium of the j^{th} source according to the following relation:

(25) Adjusted Market Share

$$\hat{q}_j(t) \propto \left(\frac{1}{p_j(t) + \delta_j(t)}\right)^{\gamma}$$

where $\delta_j(t)$ is the price premium for source j. Normalization of relation (25) so that the total quantity allocated equals the total quantity demanded gives the following relation:

(26) Static Allocation of Demand

$$\hat{q}_j(t) = q(t) \frac{(p_j(t) + \delta_j(t))^{-\gamma}}{\sum_{\text{all } j} (p_j(t) + \delta_j(t))^{-\gamma}}$$

For the case of only two inputs, Figure 11 illustrates relation (26) as a market share curve. As the figure shows, large values of γ increase the sensitivity of the allocation to price, and a very large γ would result in behavior equivalent to cost minimization. Note also that (26) assigns equal market shares when the adjusted price $p_j(t) + \gamma_j(t)$ is equal for all j sources.

The static allocation of demand in (26) assumes decision-makers reallocate demand instantly in response to changes in relative prices. In reality, the response to prices is slowed by planning and construction lead times and a natural reluctance on the part of decision-makers to change. Moreover, as described earlier, the prices computed by the simple conversion process (and other processes) reflect complete knowledge of future prices. Uncertainty about future prices would further slow and change the response to the prices used in the model.

The dynamic allocation of demand within the allocation process is accomplished by simple adjustments to the static allocations; these simple adjustments can be expressed as market penetration curves that describe how the market approaches the static market share over time (the static allocation is actually a moving target that changes as relative prices change over time). Essentially, the behavioral lag model, as it is used here, interpolates between a myopic allocation of demand that assumes market shares do not change and a perfect foresight allocation based on the relative prices of the inputs. Figure 12 illustrates the market penetration or behavioral lag curves. Because of the lower sensitivity of existing demand to prices, as compared to new demand, different behavioral lag coefficients are specified for new and existing demand. Formally, the remaining behavioral relations for the allocation process are as follows:

(27) Dynamic Allocation of New Demand

$$q_j^n(t) = q^n(t) \left[\beta_n \frac{\hat{q}_j(t)}{q(t)} + (1-\beta_n) \frac{q_j(t-1)}{q(t)} \right]$$

where the behvaioral lag parameters, β_n, represents the response in one year to the change in static market share from the previous year's actual market share. (β_n equal to one would denote instantaneous response). Figure 12 illustrates the form of (27). Similarly,

(28) Dynamic Allocation of Existing Demand

$$q_j^e(t) = q^e(t) \left[\beta_e \frac{\hat{q}_j(t)}{q(t)} + (1-\beta_e) \frac{q_j(t-1)}{q(t)} \right]$$

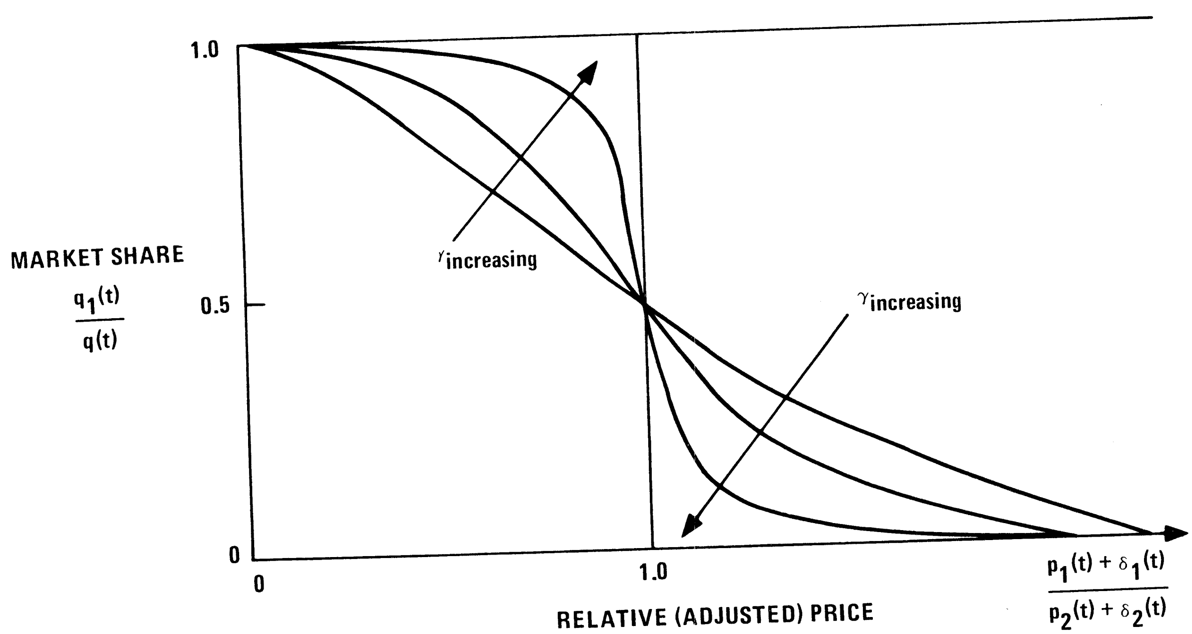

FIGURE 11. MARKET SHARE CURVE

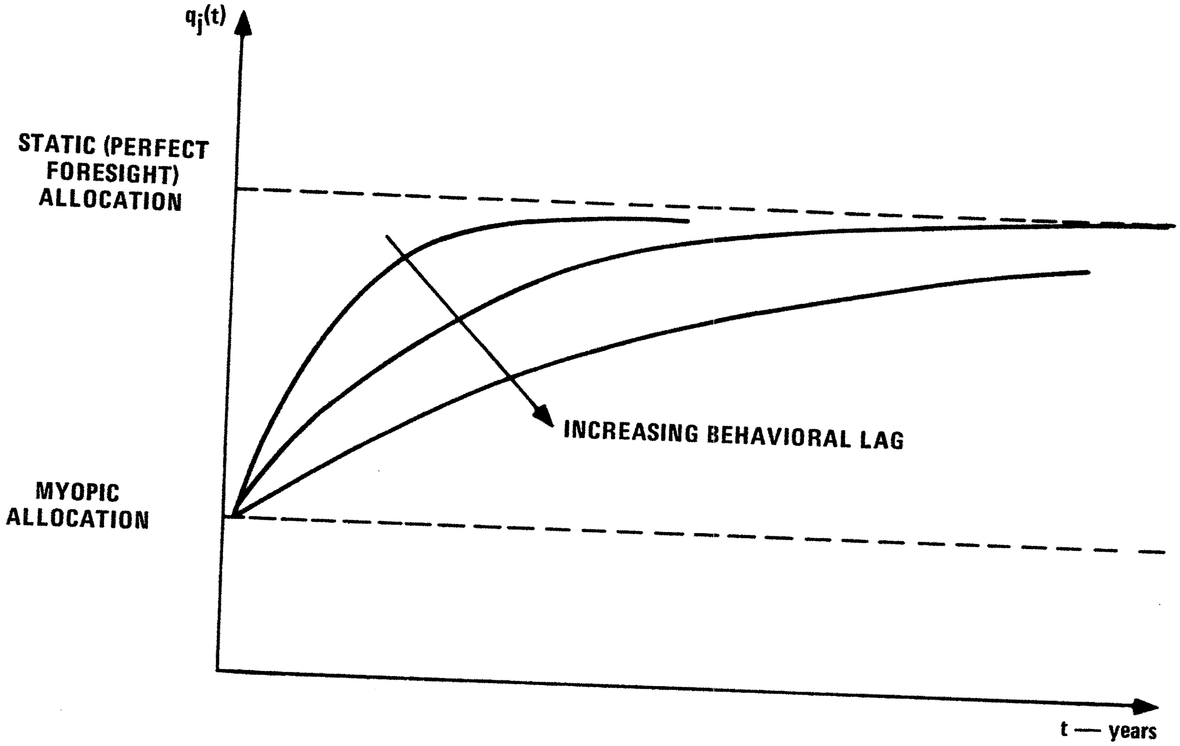

FIGURE 12. MARKET PENETRATION (BEHAVIORAL LAG) CURVE

where, typically, $\beta_e < \beta_n$. Summing the two components of demand gives the

(29) Dynamic Allocation of Total Demand

$$q_j(t) = q_j^n(t) + q_j^e(t)$$

The calculation of the prices of the outputs of a market process should reflect the relative share of each source and its price. The allocation process model further assumes that the price of the outputs of an allocation process are identical and are given by

$$(30) \qquad p_i(t) = \frac{\sum_{\text{all } j} p_j(t)\, q_j(t)}{q(t)}, \qquad \text{for all outputs, } i.$$

The allocation processes described here, when connected with the simple conversion process model, provide a dynamic model of allocation and pricing of a sector of the energy market. Of central importance in many applications of the model are the assumptions in these two processes regarding the modeling of the formation of expectations in a market. Clearly, the formation of expectations model used here is simple, yet it is not nearly so extreme as a perfect forecasting assumption or an arbitrary, myopic "rule of thumb" allocation. More complex models, including models that explicitly treat uncertainty in expectations, could be formulated within the generalized equilibrium framework if the application required such detail. The source of the parameters of the market allocation process in the current model is primarily subjective interpretation of past market share data. In future work, statistical methods applied to historical time series combined with subjective information could be used to infer the parameters of the allocation process. More discussion on this point is included in Section VI.

The allocation process is also used in close combination with the primary resource process, which is described next.

Primary Resource Process

The depletion and pricing of scarce energy resources is a unique area and the least understood aspect of energy markets. The price of a depletable resource not only depends on the cost of exploration, development and production, but also on the replacement value of the resource in competition with other energy sources in both current and future markets.

The basic information necessary to describe primary resource economics is provided by a long-run resource supply curve. Figure 13 shows the general form of a resource supply curve (example curves for crude oil were shown in Figure 6). Note that the horizontal axis is denoted by Q and is the sum of the cumulative production plus the production that is committed to the wells or mines used to meet current production. The vertical

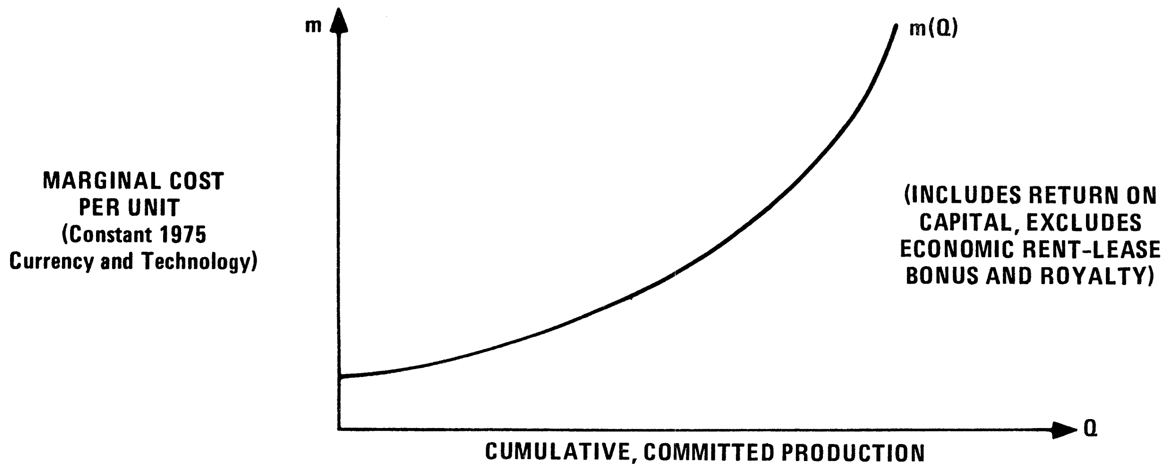

FIGURE 13. DEFINITION OF LONG-RUN MARGINAL COST CURVE FOR RESOURCE SUPPLY

axis expresses marginal cost, m(Q), assuming technology is fixed and currency is expressed in constant units as of the initial year. Thus the curve defined here is independent of time which allows time-dependent effects to be included separately. Return on invested capital is included in the cost, but price-dependent costs such as lease bonus or non-standard royalty payments are excluded from cost since the model adds these payments to cost in the computation of economic rent.

The physical relations of the primary resource process simply determine Q(t), the cumulative, committed production over time from q(t), the production, over time.

PHYSICAL RELATIONS:

(31) Cumulative, Committed Production

$$Q(t) = Q(t-1) + c_a(t) \cdot L$$

where L is the average life of a well or mine.

(32) Capacity Additions

$$c_a(t) = c(t) - c(t-1) + c_r(t)$$

(33) Capacity

$$c(t) = \max\,[q(t),\, c(t-1) - c_r(t)]$$

(34) Capacity Replacement for Depletion

$$c_r(t) = c_a(t-L)$$

(35) Initial Conditions

$$Q(0) = c(0) \cdot \frac{1}{L}\ ;\ c_r(t) = \frac{1}{2L}\ \text{For All}\ t<L$$

Figures 14 and 15 illustrate relations (31) through (35). These relations assume a mine or well has level production over a fixed life, L. In implementing relation (34), adjustments (not shown here) are made to account for unused capacity and the resulting extension in the life of a mine or well.

BEHAVIORAL RELATIONS:

The behavioral relations determine the price of a primary resource at a given production over time q(t). The computation of prices of primary resources proceeds in three steps. First, marginal cost as a function of time m(t) is determined from the long-run supply curve m(Q) and the cumulative, committed production Q(t) as given by relation (31). It is important to recognize that the long-run marginal cost curve is specified in terms of the technology and currency values for the initial period. In computing m(t), adjustments for technological change and inflation from the initial period are incorporated as shown below:

(36) Long-Run Marginal Cost

$$m(t) = m(Q(t)) \cdot f_i(t) \cdot f_c(t)$$

(37) Inflation Factor

$$f_i(t) = (1+r_{inf})^t$$

(38) Technological Change Factor [similar to Relation (16)]

$$f_c(t) = \begin{cases} 1+\rho_e(t_a-t) & t<t_a \\ f_\infty + (1-f_\infty)\,(1+\rho_c)^{t-t_a} & t\geq t_a \end{cases}$$

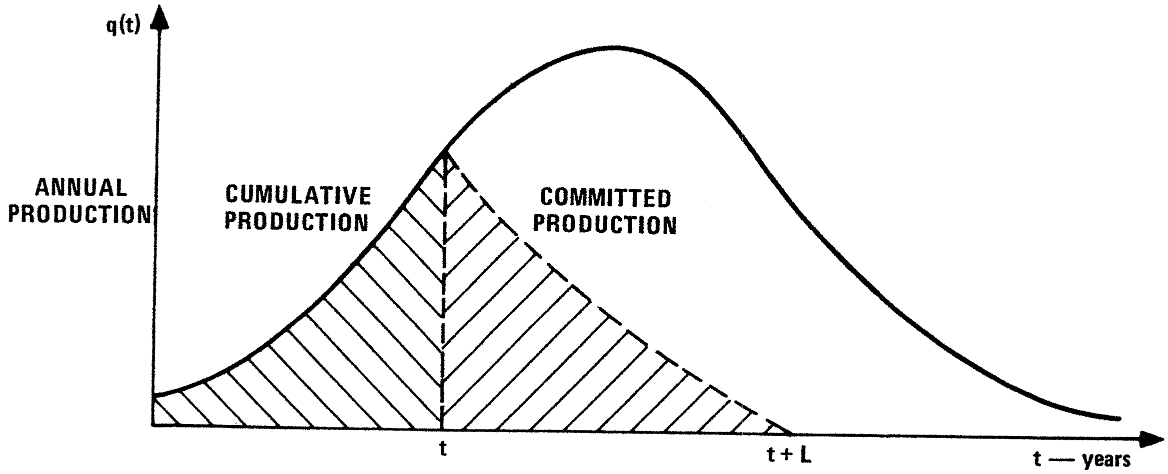

FIGURE 14. CALCULATION OF CUMULATIVE, COMMITTED PRODUCTION

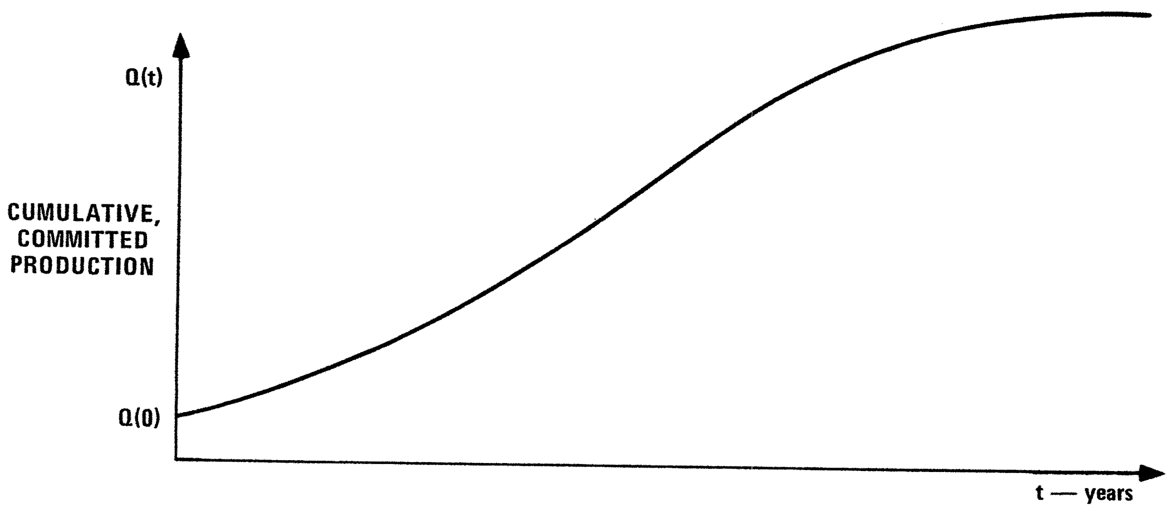

FIGURE 15. CUMULATIVE COMMITTED PRODUCTION

The second step is to compute *economic rent*, which is the difference between the marginal cost and the price of a depletable resource. Under normal conditions, the price of a depletable resource must equal or exceed marginal cost, since resource owners would have no incentive to produce at a price lower than marginal cost. In addition, the resource owner generally has the alternative of postponing investment and production in anticipation of higher prices. If r is the time preference discount rate of the resource owner, then the minimum acceptable long-run price to the resource owner is given by

(39) Long-Run Price (and Economic Rent)

$$\hat{p}(t) = m(t) + \underbrace{\max_{\tau > t} \frac{p(\tau) - m(t) \cdot f(t,\tau)}{(1+r)^{\tau - t}}}_{\text{"economic rent"}}$$

where the marginal cost is adjusted for technological change and inflation from the current period t to some future period τ by the

(40) Adjustment to Marginal Cost

$$f(t,\tau) = \frac{f_i(\tau)}{f_i(t)} \cdot \frac{f_c(\tau)}{f_c(t)}$$

Relations (36) and (39) are illustrated in Figure 16.

Relation (39) requires that the price equal or exceed marginal costs and that present and future prices of a resource be related through the discount rate and economic rent. Also note that relation (39) is computed backwards in time; future prices affect current prices. This backward computation sequence requires terminal value assumptions on $\hat{p}(t)$ for $t = T$, the terminal year. Usually, the assumption is made that the economic rent

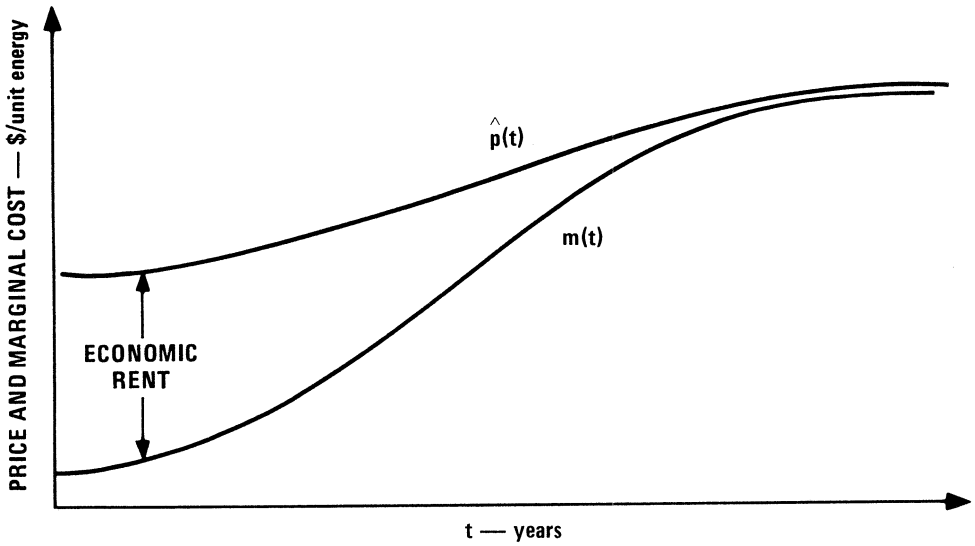

FIGURE 16. PRIMARY RESOURCE PRICES AND ECONOMIC RENT

in the terminal year is negligible if the terminal year is very distant and the competitive energy resources, such as solar or fusion, are not depletable and have small economic rent. Alternatively, a fixed level or other approximation of economic rent at the horizon year can be used.

Economic rents and prices as computed in relation (39) are relatively high in the presence of low time preference rates, high production rates, and steep resource supply curves. If near-term prices of a primary resource are higher than the alternatives, the iterative algorithm will lower near-term production on the next iteration. Lower production results in lower near-term economic rents and prices. The solution to the model thus satisfies relation (39) in addition to all other relations in the model.

Short-run supply phenomena are not modeled in (39). To adjust $\hat{p}(t)$ for short-run phenomena, we use both the secondary material concept and the growth and excess capacity multiplier concept introduced in the discussion of the simple conversion process. For a resource process, the current model uses one secondary material to represent all of the factors of production (such as surface mining equipment) used in producing a given resource or class of resources such as coal. The requirements for secondary materials for a resource process are given by

(41) Secondary Material Requirements

$$w(t) = c_a(t) \cdot m(t) \cdot s$$

where s defines the quantity of secondary materials per unit of marginal cost.

An excess capacity and growth rate multiplier I(t) are computed for a resource process using relations identical to (11), (12), (13), and (14) for the simple process. Thus, given the secondary material price, $\pi(t)$, and I(t), the adjusted primary resource price is computed as follows:

(42) Adjusted Primary Resource Price

$$p(t) = \hat{p}(t) \cdot I(t) \cdot \pi(t)$$

The primary resource process relations described here are simple because of the high value (and great difficulty) in introducing into common practice the basic concepts of economic rent and long-run supply curves. More complex implementations of a primary resource process could distinguish between investment and operating costs in developing the long-run supply curve m(Q). Implementations could also be developed that are more sophisticated in describing the expectations about the future prices used in computing economic rent. These enhancements will be important in certain applications where decisions are highly sensitive to economic rent or the capital structure of the extraction sector.

End-Use Demand Process

End-use demand is demand for energy services such as space heat or vehicle miles traveled. Unlike many energy models that use energy product demand as the final energy demand, the demand for energy products in this model is derived from projections of end-use demand. As described earlier, the competition among energy products and technologies is modeled using allocation and conversion process models. The end-use demand model is not concerned with fuel or technology competition since these are treated by end-use conversion and allocation processes; rather, it describes energy services demand as a function of population, economic activity, and price (marginal cost) of energy services.

The end-use model currently utilized in the SRI-Gulf model is relatively simple since the problems for which the model has been used to date primarily involve evaluation of supply technologies. More complex models of economic activity and its relation with energy, including the effect of changes in energy production on energy demand, are possible using the generalized equilibirum methodology.

In the current SRI-Gulf end-use demand model, growth in demand is a function of regional population and economic activity and time and price. The quantity of demand q(t) is first projected at given reference prices $p^o(t)$ using projections of population, economic activity by sector and consumer preferences. The resulting reference demand is noted by $q^o(t)$. Using an estimate of the price elasticity of demand by sector, the actual price p(t) is computed. Specifically, the end-use demand process relations are as follows:

PHYSICAL RELATIONS:

(43) Reference Demand*

$$q^{\circ}(t) = a(t) \cdot GNP(t) + b(t) \cdot POP(t)$$

where a(t) and b(t) are coefficients for a particular end-use demand sector and GNP(t) and POP(t) describe projections of regional shares of gross national product and population over time at the reference prices and for the appropriate regions.

In developing a(t) and b(t), available data and output from models of the economy and energy are used. Nothing in the methodology restricts the end-use demand model to the simple form used here and future versions of the model are expected to include more detailed representations of the economy and the many feedback effects of energy on the economy that affect energy costs.

*For example, the reference demand relation for urban residential and commercial space heat is given by:

R/C Space Heat = (Population) × (Urban Population Fraction) × (Space Heat per Urban Population) + (Population) × (GNP/Populatin) × ($ Commercial Value Added/GNP) × (Space Heat per $ Commercial Value Added)

The effect of price on energy demand is accounted for by simple constant elasticity demand curves combined with a simple lagged adjustment process. The following relation describes how the reference demands are adjusted for changes in price from the reference price trajectories:

BEHAVIORAL RELATIONS:

(44) Price Sensitive Demand

$$q(t) = q^{\circ}(t) \cdot \left(\frac{p(t)}{p^{\circ}(t)}\right)^{\nu_s} \cdot \left(\frac{q(t-1)}{q^{\circ}(t-1)}\right)^{\lambda}$$

where $\nu_S = \nu_\ell(1-\lambda)$

and ν_S = short-run (one year) elasticity

ν_ℓ = long-run elasticity

λ = lag parameter

The parameters of the end-use demand model have been determined from available statistical data and by comparison with other demand equations of other energy models. In making comparisons with demand elasticity and lag parameters of other models, care must be taken that the comparison is made in terms of prices and quantities of energy materials at the same stage of processing and transportation.

Transportation Process

A transportation process describes the economics of pipelines, unit trains, tankers, barges, or electric transmission lines between two locations defined in the model. The economics of transportation naturally depend on the distance between the locations, the costs of terminals, and the material being transported. Over a fixed distance, the relations used in a transportation process are identical to those of a simple conversion process; a transportation process involves capital costs, operating costs, and in some cases losses of the material transported (such as fuel used for pumping by a gas pipeline or line losses on electrical transmission lines). Losses of the material transported can be modeled in the same way that the thermal efficiency of a conversion process is modeled.

Starting with the relations defined for the simple conversion process, it is only necessary to adjust the parameters of the simple process for the length of a transportation link. In the model, the economics of each actual or potential transportation route and mode can be specified individually. To economize on the amount of input data to the model and assure consistency, the economics of a transportation link over a reference distance is specified and simple adjustment formulas are used to adjust the capital, operating costs, and thermal efficiency for changes from the reference distance. Capital costs are adjusted as follows:

(45) Transportation Capital Cost Adjustment

$$k(t) = k'(t) \cdot b_k$$

such that $$b_k = \left[a_k + (1-a_k) \frac{d}{d_o} \right]$$

where k'(t) is the capital cost for a transportation link over distance d_o, d is the distance over the actual link and a_k is the fraction of the capital cost over distance d_o that is fixed (such as terminal costs). Similarly,

(46) Transportation Operating Cost Adjustment

$$v(t) = v'(t) \cdot b_v$$

(47) Transportation Thermal Efficiency Adjustment

$$e = (e')^{b_e}$$

where b_v and b_e are defined analogously to b_k.

In using these relations to model a national transporation network with many links and terminals it is necessary to aggregate to a network with far fewer links and terminals. In such an aggregated network, terminal and line costs tend to be indistinguishable and as a result the fixed fractions (a_k, a_v, a_e) tend to be small. For this reason, the fixed fractions are set to zero in the current model and special cases such as Alaskan pipelines and LNG (liquified natural gas) terminals and ships are given parameter estimates that do not rely on the distance adjustment relations.

The complete transportation network for high Btu gas is shown in Figure 3; other fuels have similar transportation networks defined in the model. The transportation processes, in concert with the other processes in the model, simultaneously determine capacity and flows along all of the routes in the transportation network. In determining these flows, the overall economics and other behavioral factors associated with all model processes are considered. The great sophistication of the model in allocating transportation flows can be illustrated by considering how the model would operate as an optimization transportation model. If all the allocation processes were set to select the minimum cost alternative source, the model would find the minimum cost transportation network capacity and flows as well as conversion and production capacity and flows for the model.

Complex Conversion Process

A complex conversion process is a multiple input or multiple output conversion process used to model more complex conversion industries such as electric power generation, refining, joint production of natural gas and natural gas liquids, or end-use processes such as a heat pump that may satisfy both air conditioning and space heating loads. Usually, complex conversion processes can be constructed by using a subnetwork of simple conversion processes or by making minor changes to the simple conversion process logic. Conceptually, any well-defined submodel of a complex conversion process could be included in the overall model network by simply interfacing the computer code and data for the submodel with the appropriate input and output variable definitions and computer memory linkages.

The following four types of complex conversion processes will be described here: electric power generation, refining, auxiliary input/output processes, and joint natural gas and LPG (liquid petroleum gas) production processes.

a) *Electric Power Generation*

Electric power generation is complicated by the need for generation to follow a fluctuating demand and by the relatively high costs of storing electric energy as compared to other forms of energy. The demand for electric energy can be characterized in terms of the quantity of energy per year (kilowatt-hours) that must be generated at various given rates or power levels (kilowatt-hours/hour). Peak power levels tend to occur a small fraction of the time and a base power level must be provided continuously. The most efficient (low operating and fuel cost) generation technologies tend to be the most capital-intensive. Thus electric power systems use a mix of technologies with different efficiencies and capital costs to satisfy the fluctuating demand in the most economic way.

The electric power process in this model characterizes electric power demand in terms of a discrete number of electric demands at various power levels. Each electric demand category is distinguished by its load factor LF_i. Typical load factors are 0.13 for peak, 0.40 for cycling, and 0.70 for base load demand. These electric demands can be represented in the energy model network as distinct materials. Depending on the resolution required, any number of power levels or electric materials could be represented. For a general national model, three materials representing base, cycling, and peaking power load categories is usually sufficient.

Figure 17 shows a simplified version of the subnetwork used to model electric power generation. This subnetwork is composed of an electric power load disaggregation process, an allocation process for each load category and several electric power conversion

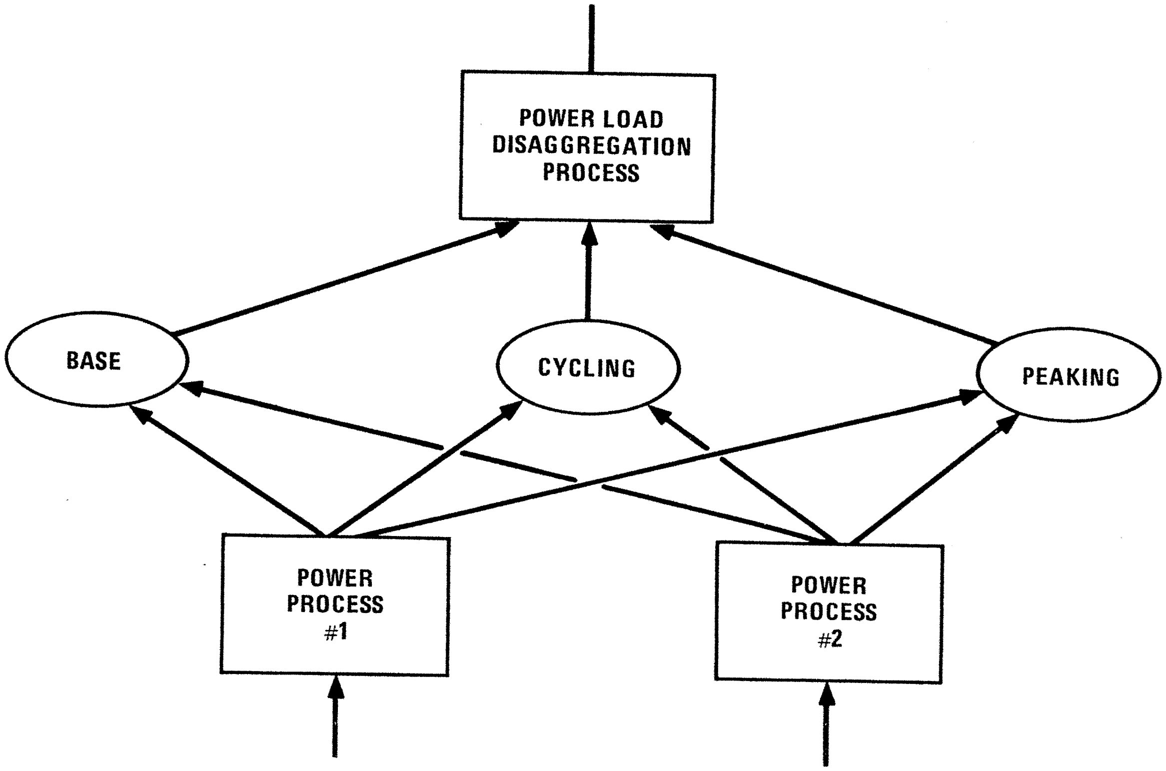

FIGURE 17. ELECTRIC POWER SUBNETWORK

processes. A more detailed illustration of the electric power subnetwork for one of the nine demand regions, containing more than twenty conversion processes, is shown in Figure 5.

The power load disaggregation process simply disaggregates the demand for electric power into three categories of demand: base, cycling and peaking demand, and computes the price of electricity from the prices and quantities of base, cycling and peak electricity. The relations for the power load disaggregation process are as follows:

PHYSICAL RELATIONS (Disaggregation Process):

$$(48) \qquad q_j(t) = a_j(t)\, q(t) \qquad j = 1, \ldots, J$$

where $a_j(t)$ is the fraction of total electric energy in load category j. Typical values of $a_j(t)$ are 0.05 for peak, 0.25 for cycling and 0.70 for base load. These energy fractions should not be confused with the load factor.

BEHAVIORAL RELATIONS (Disaggregation Process):

$$(49) \qquad p(t) = \sum_{j=1}^{J} a_j(t)\, p_j(t)$$

In some applications of the model, for example the evaluation of peak load pricing policies, it would be necessary to model electric power transmission and distribution in a way that retains the disaggregated electric power load categories. This could be accomplished within the network language without difficulty.

The allocation processes shown in Figure 17 attempt to describe decisions made by power system planners in expanding and operating power systems for each load category. Using the same relations as the allocation process described earlier the allocation process for each load category selects a mix of power conversion processes by allocating demand among the processes based on the marginal cost (price) of electricity from each conversion process in each load category. As will be shown, the marginal costs of generation reflect capital costs and present future operating costs of generation and also reflect the opportunity cost of a generating process that could be used to satisfy other load categories.

The electric conversion technologies shown in Figure 17 are similar to the simple conversion process except that the single output of the simple process is replaced by multiple outputs. Each output of an electric power conversion process describes the marginal cost (price) of power at the load factor describing the load category. The relations for the electric power conversion process draw on those of the simple conversion process.

PHYSICAL RELATIONS (Electric Conversion Process)

(50) Total Output

$$q(t) = \sum_{\text{all } i} q_i(t)$$

(51) Capacity

$$c(t) = \sum_{\text{all } i} \frac{q_i(t)}{LF_i(t)}$$

where $LF_i(t)$ describes the load factor (power level in kwh/hr) for the i^{th} output. Since peak load factors are smaller than cycling load factors, which are in turn smaller than base load factors, the capacity required to produce a given quantity of energy is far larger for peak load than for cycling load and larger for cycling load than base load. The remaining physical relations for the electric conversion are identical to relations (1) through (6) for the simple conversion process.

BEHAVIORAL RELATIONS (Electric Conversion Process):

(52) Product Pricing

$$p_i(t) = \phi(t) + \left[\frac{k_d(t)}{1-f_{tax}} - \sum_{\tau=t+1}^{t+L} \frac{p(\tau) - \phi(\tau)}{(1+r)^{\tau-t}} \right] I(t) \frac{1}{LF_i(t)}$$

All other behvaioral relations for the simple conversion process apply. The reliability of facilities and the need for reserve capacity are reflected in the capital costs, $k_d(t)$, and the load factors, LF_i. As with the simple process, the product prices reflect future costs, prices, and other factors.

The functioning of the entire electric power subnetwork can best be illustrated by considering what would happen if the parameters of the allocation processes in this subnetwork were adjusted so as to choose minimum cost allocations of demand to power conversion processes. The resulting optimization model would consider both intertechnology and intertemporal tradeoffs in selecting the minimum discounted cost expansion pattern. For example, the resulting optimization model would distinguish between a base load plant that would continue in base load operation and another plant that would later satisfy cycling or peaking demand. A detailed electric capacity-expansion model using this approach is described in [7] and [11]. In the SRI-Gulf model, the allocation process parameters are set to reflect lags in decision-making resulting from uncertainty and regulation (behavioral lag) and the desire for diversity in power generation processes (market share curve). Thus the model is more descriptive of actual electric power industry decisions than a pure optimization model.

The current version of the model uses marginal cost pricing on electricity, a departure from current practice; however, over the fifty year horizon of the model the difference between marginal and rate base pricing is relatively small.*

b) *Refinery Processes*

A crude oil refinery produces liquid fuels such as gasoline, distillate fuel oil and residual fuel oil. Modeling of refining economics is complicated because the relative costs of refinded products depend on the mix of products produced. Refining costs, however, represent only about ten percent of the cost of distributed refined products and therefore an approximate refining process model is adequate for most applications.

The subnetwork for the refinery process used in the present SRI–Gulf model is shown in Figure 18. The three primary outputs of the refinery process are residual fuel oil, distillate fuel oil and gasoline. As shown by the dotted line in the figure, demands for other refined products such as marine bunker fuel, jet fuel and naptha feedstock are characterized in terms of demand for the three primary products. This aggregation across product types allows the refinery process model to focus on important differences in economics among the three primary products and overall refining costs rather than detailed modeling of price differentials among a large number of products.

The refinery loading subprocess shown in Figure 18 allocates the demand for products among three types of refineries considered in the model and computes the prices of residual fuel oil, distillate fuel oil, and gasoline using the prices of refined products from each of these refineries. The three types of refineries in this model represent low, medium, and high gasoline fraction refineries. This characterization of refineries by the gasoline fraction of products reflects the dependence of refined product prices on the relatively large investment and operating costs required to achieve high gasoline fractions. In problems where additional resolution of refinery costs is necessary, additional types of refineries reflecting different product mixes can be defined. If this did not give sufficient resolution, a detailed structural model of refinery flows and components could be developed within the generalized equilibrium framework.

The computation of the prices of refined products from each of the refinery types is computed by simple process models. The allocation process for low gasoline fraction refineries selects between front end

*Rate base pricing of electric power is being included in new versions of the model currently under development.

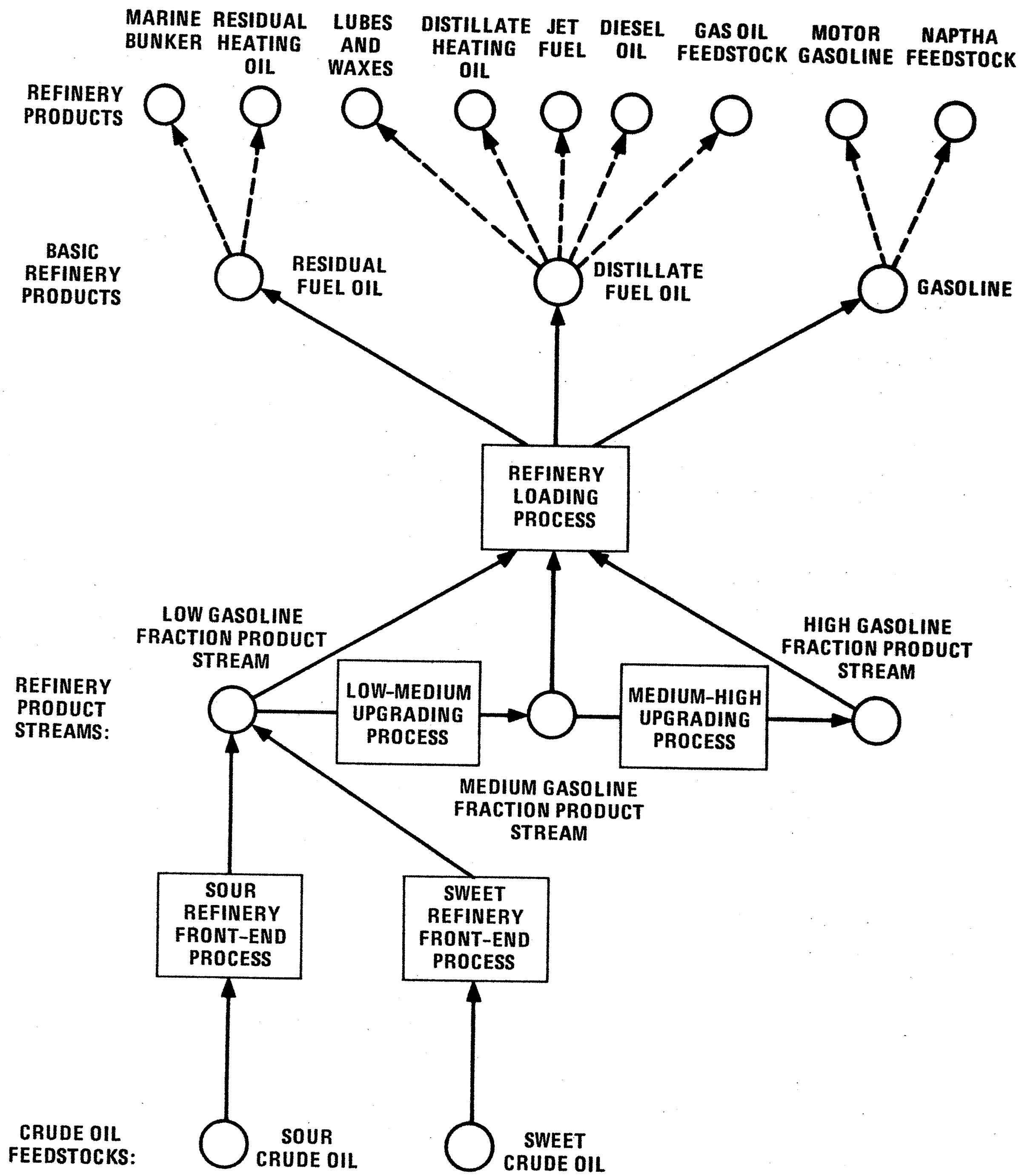

FIGURE 18. REFINERY PROCESS NETWORK STRUCTURE

refinery facilities based on sour (high sulfur) or sweet (low sulfur) crude oil. The additional investment and operating cost to construct a medium gasoline fraction refinery is modeled using the low-medium upgrading simple process. Similarly the production economics of high gasoline fraction products are related to the economics of medium gasoline fraction products by the medium-high upgrading simple process.

The overall refinery model therefore describes the economics of a refinery industry that draws on a range of crude oil as represented by sweet and sour crude oils and produces a range of products as represented by gasoline, distillate, and residual fuel oil.

Since the refinery model uses the simple conversion process relations described earlier, only the relations imbedded within the refinery loading model need be documented here.

PHYSICAL RELATIONS (Refinery Loading Process):

(53) Refined Product Fractions

$$f_g(t) = q_g(t)/q(t)$$

$$f_d(t) = q_d(t)/q(t)$$

$$f_r(t) = q_r(t)/q(t)$$

where the subscripts g, d, and r refer to gasoline, distillate fuel oil, and residual fuel oil, and

(54) Total Refined Product Demand

$$q(t) = q_g(t) + q_d(t) + q_r(t)$$

Allocation of refined product demand among the three types of refineries is by interpolation based only on the gasoline fraction. Let F_g^h, F_g^m, and F_g^ℓ denote the gasoline fraction describing the high, medium, and low gasoline fraction refinery types and let $q_h(t)$, $q_m(t)$ and $q_\ell(t)$ be the total of all refined products produced by each refinery type. The interpolation must recognize the following four cases:

(55) Refinery Production by Type

Case 1: $f_g < F_g^\ell$

$$q_\ell(t) = q(t);\ q_m(t) = 0,\ q_h(t) = 0$$

Case 2: $F_g^\ell \leqslant f_g < F_g^m$

$$q_\ell(t) = q(t)\,\frac{f_g(t) - F_g^m}{F_g^\ell - F_g^m};$$

$$q_m(t) = q(t) - q_\ell(t);\ q_h(t) = 0$$

Case 3: $F_g^m \leqslant f_g < F_g^h$

$$q_\ell(t) = 0;$$

$$q_m(t) = q(t)\,\frac{f_g(t) - F_g^h}{F_g^m - F_g^h};$$

$$q_h(t) = q(t) - q_m(t)$$

Case 4: $F_g^h \leqslant f_g$

$$q_\ell(t) = 0;\ q_m(t) = 0;\ q_h(t) = q(t)$$

BEHAVIORAL RELATIONS (Refinery Loading Process):

Pricing of the outputs of the refinery loading process is based on the prices of refined products, e.g., p_g^h, that are inputs to the loading process.

(56) Prices of Products

$$p_g(t) = \frac{p_g^h(t)\, q_h(t) + p_g^m(t)\, q_m(t) + p_g^\ell(t)\, q_\ell(t)}{q(t)}$$

$$p_d(t) = \frac{p_d^h(t)\, q_h(t) + p_d^m(t)\, q_m(t) + p_d^\ell(t)\, q_\ell(t)}{q(t)}$$

$$p_r(t) = \frac{p_r^h(t)\, q_h(t) + p_r^m(t)\, q_m(t) + p_r^\ell(t)\, q_\ell(t)}{q(t)}$$

where the prices of refined products, e.g., $p_g^h(t)$, for each refinery type are given by fixed ratios, e.g., r_g^h, to the average prices of refined products, e.g., $\bar{p}_h(t)$, for each refinery type as follows:

(57) Prices of Products by Types of Refinery

$$p_g^h(t) = r_g^h\,\bar{p}_h(t)\ ;\ p_d^h = r_d^h\,\bar{p}_h(t)\ ;\ p_r^h = r_r^h\,\bar{p}_h(t)$$

$$p_g^m(t) = r_g^m\,\bar{p}_m(t)\ ;\ p_d^m = r_d^m\,\bar{p}_m(t)\ ;\ p_r^m = r_r^m\,\bar{p}_m(t)$$

$$p_g^\ell(t) = r_g^\ell\,\bar{p}_\ell(t)\ ;\ p_d^\ell = r_d^\ell\,\bar{p}_\ell(t)\ ;\ p_r^\ell = r_r^\ell\,\bar{p}_\ell(t)$$

The product price ratios are provided as parameters to the model and the average product prices by type of refinery, $\bar{p}_\ell(t)$, $\bar{p}_m(t)$, $\bar{p}_h(t)$ are computed by the three types of refinery process models that are modeled as simple conversion processes.

To account for variations in the distillate and residual fuel oil fractions, $f_d(t)$ and $f_r(t)$, from normal fractions for each type of refinery, further adjustments to the refined prices are required. If the residual fuel oil fraction is lower than normal, it is assumed distillate is mixed with residual fuel oil and the product prices are adjusted to retain the same average product price. Similarly, if the distillate fraction is higher than normal for a given fraction of gasoline demand, gasoline is diverted to satisfy distillate demand and the prices are rebalanced. Finally, if in relation (55) case 4 is active, the price of gasoline is increased substantially to force the iterative algorithm to select gasoline demands within the range of operation of the three types of refineries.

c) *Auxiliary Input Conversion Process*

Processes that consume more than one input in fixed proportions to the output can be modeled using the auxiliary input conversion process. The following minor modifications to the simple conversion process are required:

PHYSICAL RELATIONS:

(58) Physical Flow (replaces Relation 1)

$$q_{fj}(t) = q(t)\frac{1}{e_j}$$

where e_j is the quantity of output product per unit of input product j. Any number of inputs could be allowed.

BEHAVIORAL RELATIONS:

(59) Operating Cost (replaces Relation 8)

$$\phi(t) = v(t) + \sum_{j=1}^{J} p_{fj}(t)\frac{1}{e_j}$$

where $p_{fj}(t)$ is the price of the j^{th} input or feedstock. The auxiliary input process is used in the present model, for example, to model automobiles that consume a mixture of gasoline and methanol.

d) *Joint Product Process*

Multiple output processes where the outputs must be produced in fixed proportions can be modeled as joint product processes. A joint product process builds on the simple conversion process or resource supply process; the additional relations for a joint product process are as follows:

PHYSICAL RELATIONS:

(60) Total Output

$$q(t) = \sum_{\text{all } j} q_j(t)$$

BEHAVIORAL RELATIONS:

(61) $$p_j(t) = p(t) + \lambda_j$$

where λ_j is a price premium (positive or negative) added to the price (marginal cost) of total products in order to force the quantities demanded to satisfy the following relation:

(62) Output Ratio Constraints

$$r_j = \frac{q_j(t)}{q(t)}$$

The price premium λ_j can be interpreted as a shadow price on the output ratio constraint. The price premium is set by an iterative adjustment process that is described in Section V. An example of a joint product process is the joint production of LPG (liquid petroleum gases) and natural gas where the ratio of LPG to natural gas is about one-to-ten.

Price and Quantity Regulation Processes

Government intervention in energy markets often takes the form of controls on energy prices and quantities. In some cases, such as rate base pricing of electric utilities, the regulation allows supply and demand to balance by allowing a reasonable return on investment with protection from competition. In other cases of government regulation, rationing or other non-market allocation processes are required to bring supply and demand into balance. For example, natural gas price regulation can result in shortages of gas that require allocation of available supply by a public agency.

Implementation of price and quantity regulation in a generalized equilibrium model requires additional relations describing the regulatory restrictions and associated shadow prices. The shadow prices can be used to model the allocation or rationing decision of government in response to excess demands. The following types of relations are used in a process to model regulation of various forms:

PHYSICAL RELATIONS:

(63) Quantity Regulation (Equality)

$$q(t) - q^*(t) = 0$$

(64) Quantity Regulation (Inequality)

$$q(t) - q^*(t) \leqslant 0$$

(65) Price Regulation (Equality)

$$p(t) - p^*(t) = 0$$

(66) Price Regulation (Inequality)

$$p(t) - p^*(t) \leqslant 0$$

In each case the term denoted by the * is the constrained value of the variable.

Of the constraint relations shown here, the inequality quantity regulation and inequality price regulation are most meaningful. An example of an inequality quantity regulation is an import quota on crude oil. An example of inequality price regulation is natural gas price controls.

For each constraint relation, a shadow price must be provided in addition to the market prices. Shadow prices are set by the network iteration algorithm and are adjusted up or down until the constraint is satisfied (similar to the joint product process shadow price adjustment). This iterative adjustment process is described in Section V.

Shadow prices have their impact through their effect on allocation processes. However, we must now distinguish between allocation processes based on rationing and those based on market prices. The previously described allocation process suffices for market processes; a simple rationing model can be implemented by modifying the market share relations of the allocation process to operate on shadow prices as follows:

BEHAVIORAL RELATIONS:

(67) Allocation Process Market Share Relations (replaces Relation 26)

$$\hat{q}_j(t) = q(t) \frac{(\lambda_j+\delta_j)^{-\gamma}}{\sum_{\text{all}} (\lambda_j+\delta_j)^{-\gamma}}$$

The market prices and shadow price output from an allocation model are weighted prices computed as in relation (30).

The interpretation of the market share relation using shadow prices is that of a rationing process that tends to allocate supply to the purchasers who would benefit the most and allocate demand to those suppliers whose costs are lowest or whose products are demanded less by others.

In the situation where the input to a conversion or transportation process is a constraint (has a nonzero shadow price), two cases must be considered. First, if the conversion process pricing is unregulated, then the market price of the output product will equal its shadow price. Second, if the conversion process pricing is regulated, the shadow price on the input will be passed through the process with appropriate adjustment for thermal efficiency. The behavioral relations shown below illustrate these two cases:

(68) Conversion Process (Unregulated)

$$\lambda(t) = \hat{p}(t) + \mu(t) + \frac{\lambda_f(t)}{e}$$

$$p(t) = \lambda(t)$$

(69) Conversion Process (Regulated)

$$\lambda(t) = \hat{p}(t) + \mu(t) + \frac{\lambda_f}{e}$$

$$p(t) = \hat{p}(t)$$

where $\hat{p}(t)$ is the product price computed in the absence of regulation and $\mu(t)$ is the shadow price associated with any constraint imposed on the price and quantities associated with the process.

The relations shown above can be used in various combinations to represent import quotas, price regulation, utility rate base price regulation (a formula is required to specify the regulated price in relation to investment and operating costs), and environmental controls limiting outputs of pollutants (the relation between process output and pollutant output must be provided). Regulatory models must be implemented with great care since whether a conversion process is regulated or unregulated can have major effect on the allocation of energy demand and the allocation of economic surplus [quantity times the difference between the shadow price $\lambda(t)$ and the price $p(t)$]. This economic surplus may show up as profits to specific industries or technologies and thus can greatly affect the evaluation of a new technology such as synthetic gas in the presence of natural gas price regulations. In cases where the regulation is only partially effective or favoritism is shown by the regulators towards specific customer classes, the relations described here can easily be modified by interpolation among relations for regulated and unregulated processes. †

† Price regulation in the 1976 version of the SRI-Gulf model is not used in the full generality described here. In specific applications, however, regulations have been introduced into the model using the concepts discussed here. A new version of the computer implementation of the model that will permit greater flexibility in this area is under development at Decision Focus, Inc.

A pitfall in modeling regulation is that regulations often change in response to economic and other pressures. Realistic modeling of regulation over long periods of time thus requires a behavioral model of the regulatory process which could, of course, be implemented within this methodology, as a new type of process.

Another pitfall is to apply constraints to quantities or prices in order to force preconceived outputs from the model. In cases where the output of the model does not agree with intuitive reasoning, it is much better to identify the basis for the differences and, if necessary, correct the process relations or data responsible for the differences. Otherwise, constraints will tend to be used freely in a modeling effort to avoid dealing with fundamental issues and much of the insight-generating value of the model will be lost.

Secondary Materials Process

Secondary materials in the model are factors of energy production, conversion and transportation. Examples of secondary materials in the model are surface mining equipment for coal and shale processes, construction equipment and labor for refining and synthetic fuels, and large pressure vessels for nuclear reactors and coal liquification.

The secondary materials process describes the changes in prices of secondary materials resulting from changes in demands for the secondary material. For example, if the demand for pressure vessels is suddenly increased because of a sudden increase in the rate of growth of nuclear facilities using pressure vessels, the secondary materials model will increase the price of pressure vessels to all nuclear and coal liquification processes that use pressure vessels. The increase in secondary materials cost will flow through to plant capital costs and product prices. These higher prices will discourage — through the iterative solution of the model — the sudden increases in nuclear capacity that produced the higher secondary material demand. The result is capacity-expansion rates that take account of impacts on secondary markets and therefore exhibit less rapid fluctuation than would result if secondary markets were ignored by the model.

The secondary materials process can be viewed as a simplified representation of the interaction of the energy system with the rest of the economy. Thus in Figure 1, the secondary materials process would be one of the models of the "rest of the world" that are important but not crucial to the exogenous decision problems for which this model was designed.

A simplifying convention included in the secondary materials process model currently implemented is to include in the capital cost estimates the long-run trends in secondary material costs. Thus the inflation and technological change relations for the simple conversion process and the primary resource process are defined to take long-run trends in labor and equipment costs into account. This convention thus allows the secondary materials model to focus on the dynamics of the price adjustments resulting from secondary material demands.

PHYSICAL RELATIONS:

The physical relations for a secondary material simply compute the total secondary materials demand as follows:

(70) Total Secondary Materials Demand

$$w(t) = \sum_{j=1}^{J} w_j(t)$$

where $w_j(t)$ is the secondary material quantity demanded by the j^{th} process.

BEHAVIORAL RELATIONS:

The behavioral relations are more complex. The secondary material price is based on a measure of the secondary material demand relative to nominal secondary industry capacity. This relation is illustrated in Figure 19 and stated as follows:

(71) Secondary Material Prices

$$\pi(t) = M(R(t))$$

In this relation R(t) is the ratio of demand to capacity defined in relation (72) below and M(R) is the secondary materials price function illustrated in Figure 19. Since by convention the nominal cost of secondary materials is reflected in the capital cost of a plant, the price function M(R) is expressed as a multiple of the nominal price where M(0) = 1.0. The shape of the secondary material function is adjusted to reflect observed industry behavior and the size of the total industry relative to the energy sector demand. The ratio of secondary material demand w(t) to capacity $\tilde{w}(t)$ is given by the following relation:

(72) Secondary Material Demand to Capacity Ratio

$$R(t) = \frac{w(t)}{\tilde{w}(t)}$$

Nominal secondary industry capacity is determined by a simple model that assumes capacity responds to increases in demand for secondary materials, but only with a time lag to allow for planning and decision-making and actual construction. The following simple expoential smoothing formula is used for this purpose:

(73) Secondary Industry Capacity

$$\tilde{w}(t) = \alpha w(t) + (1-\alpha)\, \tilde{w}(t-1)$$

where α is the response of the system capacity to demand. The half-life response time of relation (73) to a step change in w(t) is given by

(74) Secondary Industry Half-Life Response Time

$$H = \frac{\ln(0.5)}{\ln(1-\alpha)}$$

which is a useful way of characterizing secondary industry planning and construction dynamics in a simple aggregate way. Finally, the initial secondary industry capacity is specified by

(75) Initial Secondary Industry Capacity

$$w(0) = w_o$$

The overall response of the secondary material to price changes in primary industry capacity is illustrated by the series of flows in Figure 20. At the top in Figure 20(a) primary industry capacity is growing over time, but with a much higher growth rate, shown in Figure 20(b), during the middle years of the time horizon shown. In other words, primary industry capacity additions are roughly proportional to the primary industry growth rate; the specific relations for computing secondary materials demands are given by the physical relations for the simple conversion and primary resource processes. In Figure 20(c) the secondary material demand is shown to be proportional to primary industry capacity additions and secondary industry capacity is shown to respond to demand but with a lag introduced for planning and construction. At each point in time, the relative positions of the solid demand line in Figure 20(c) and the broken capacity line illustrate the demand to capacity ratio that is used to compute secondary material price in the relation illustrated in Figure 19. The resulting secondary material price over time is illustrated in Figure 20(d). Clearly, the changes in prices in Figure 20(d) would tend to discourage the primary industry capacity-expansion pattern indicated in Figure 20(a). However, even at the equilibrium of the model, patterns of prices and quantities such as those illustrated in Figure 20 can be present and still satisfy all of the relations of the model.

An alternative approach used in several other energy models [12, 13, 14, 15, 16] is to constrain the rate of growth within specified upper limits. We will discuss such approaches in Section VI, but it should be clear that such growth constraints are quite arbitrary and can result in expansion patterns for technologies that are simply dictated by these constraints. The secondary materials process described here essentially replaces these constraints with a more flexible structure that allows the secondary industry capacity to be responsive to market forces rather than imposed arbitrarily.

Finally, it is worth emphasizing that the purpose of the secondary materials model described here is not to study secondary materials problems but to build into the model the effect of short-run considerations on long-run prices and quantities of energy materials required for analysis of R&D and other strategic energy policy decisions. If the focus of the model were to analyze short-run capacity-expansion problems then a much more detailed model of secondary industries would be necessary.

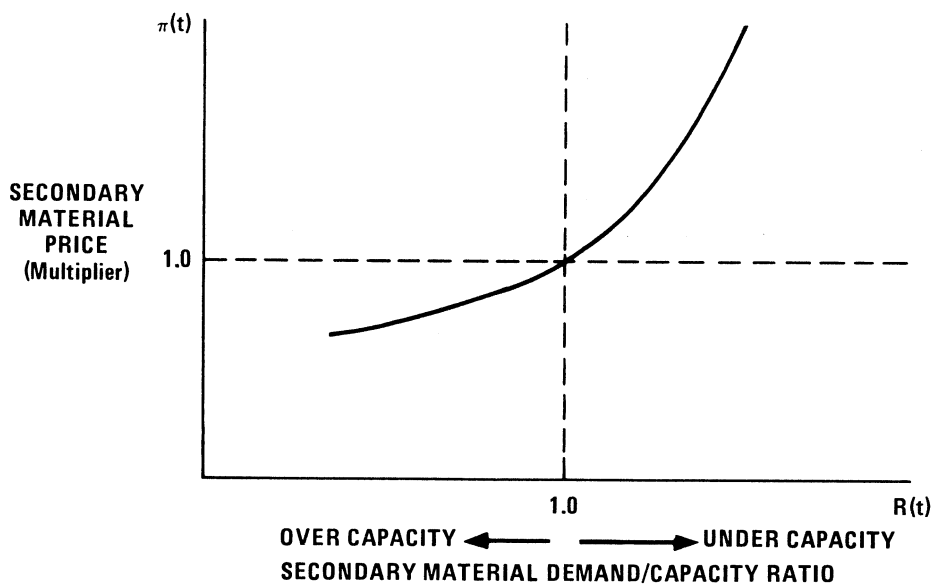

FIGURE 19. SECONDARY MATERIAL PRICE FUNCTION

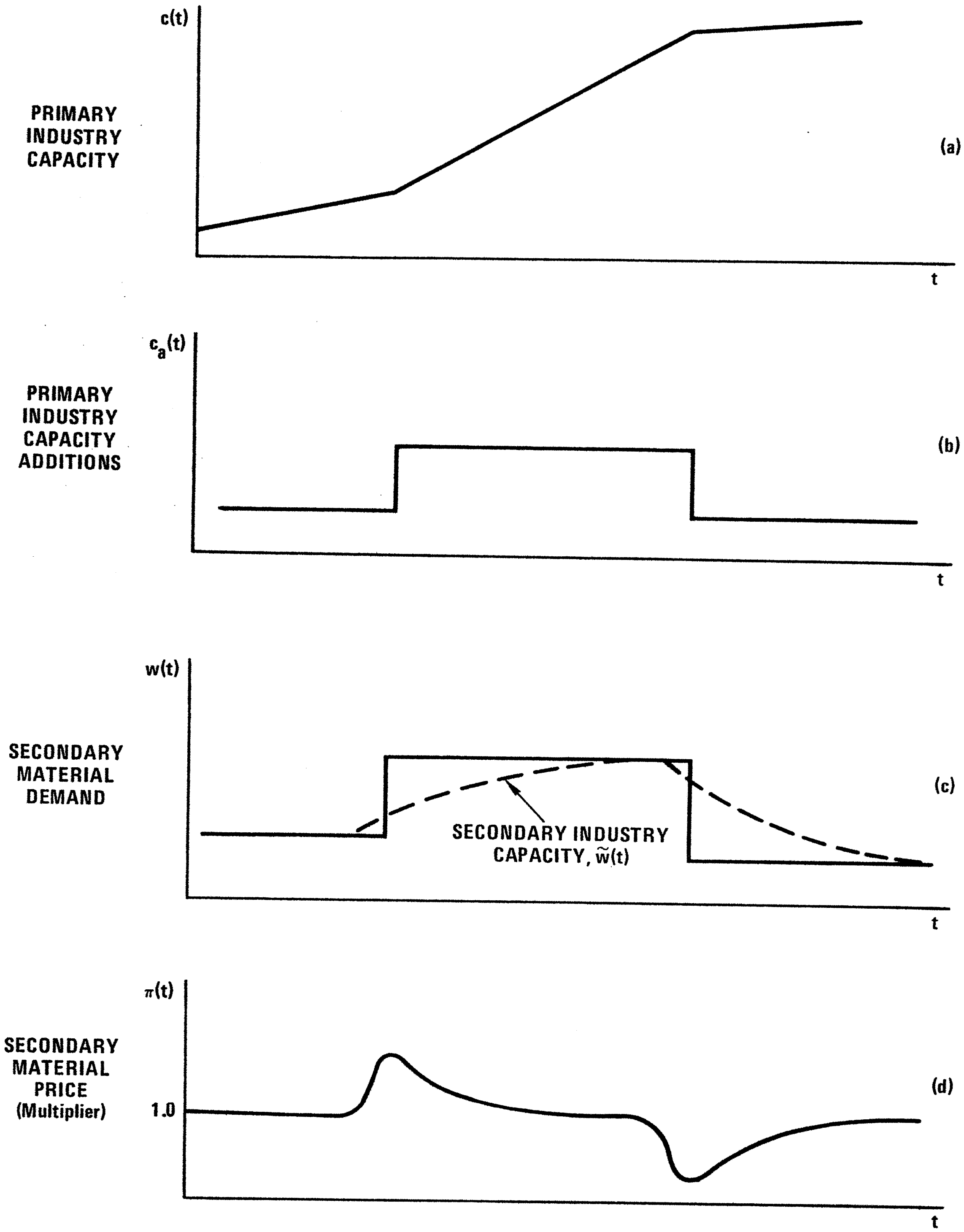

FIGURE 20. SECONDARY MATERIAL PROCESS RESPONSE

V. NETWORK ITERATION ALGORITHM

Finding the solution to a network of processes containing physical and behavioral relations such as those just described is accomplished using the iterative algorithm summarized in Section III. Figure 21 describes the algorithm in more detail. Note in particular how the algorithm takes advantage of the dynamic structure of the process relations by solving physical relations in forward time and behavioral relations in backward time.

Relaxation Methods

The iterative algorithm described in Figure 21 involves generating tentative price and quantity estimates in a way that converges to an equilibrium solution of the model that satisfies all of the model's process relations. The algorithm, as described thus far, can exhibit several modes of behavior. It may converge in relatively few iterations to a solution; it may converge slowly to a solution; it may oscillate back and forth among solutions while converging slowly to a solution; or it may oscillate while diverging further from a solution. Often several modes of behavior of the algorithm will be present in different parts of the network. For those familiar with the solution of iterative models, such behavior is not unusual for such a simple algorithm. In this section, the modifications to the algorithm, called relaxation methods, will be described.

Relaxation methods simply interpolate among successive solutions of the algorithm in order to speed up the convergence. In most applications, the typical behavior of the algorithm on parts of the network is to oscillate. By interpolating among successive estimates of the oscillating prices and quantities, the interpolated or relaxed tentative estimates are usually closer to the ultimate solution, thereby greatly improving convergence.

The large size of the SRI-Gulf model requires that its relaxation methods not require extensive computer storage or require significant additional computations. Therefore, the approach used is to take maximum advantage of the structure of the model and the analyst's knowledge of the nonlinearities of the model. By experimentation, relaxation methods are modified until reasonable convergence is achieved, and those described in this section are the result of experimentation on the current model.

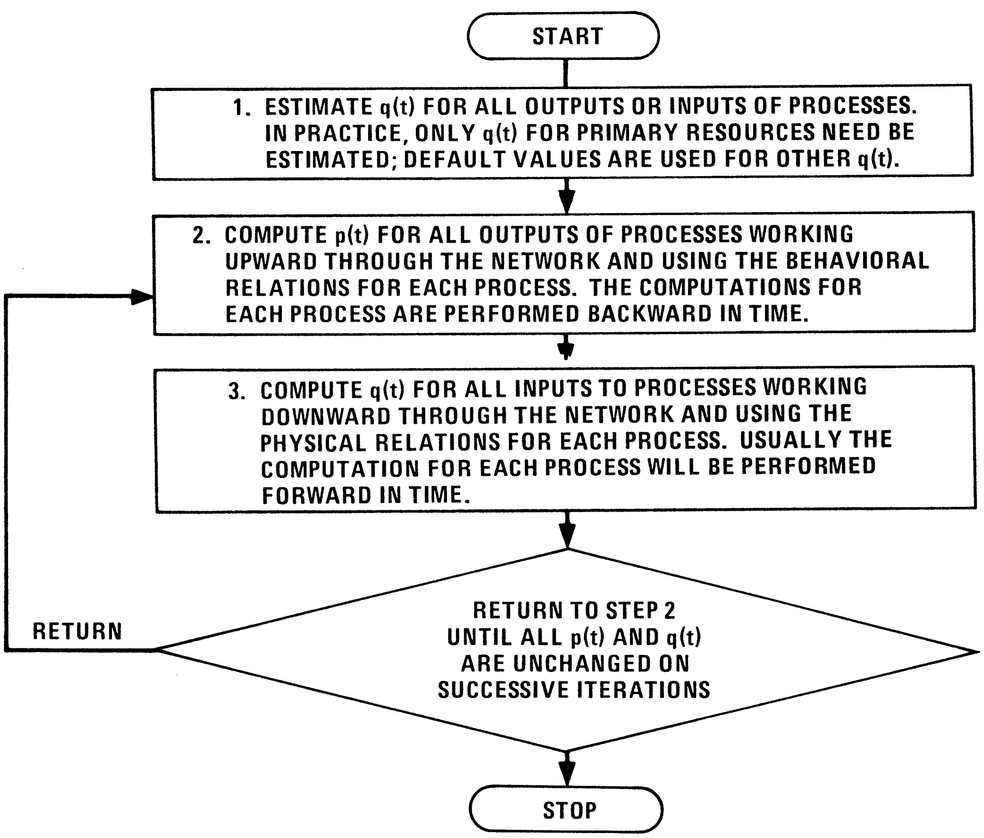

FIGURE 21. DETAILS OF THE NETWORK ALGORITHM

The most difficult sector of the model from a convergence point-of-view is the primary resource sector. For a primary resource process, the relaxation method is as follows:

$$p^n(t) = \omega^n(t)p^*(t) + (1-\omega^n(t))p^{n-1}(t)$$

where $p^n(t)$ is the price of a primary resource passed up the network on the n^{th} iteration, given the price $p^*(t)$ computed using the behavioral relations on the same iteration (using $q^n(t)$).† The relaxation coefficients, $\omega^n(t)$, are typically between 0.01 and 0.5 and are specific to the process, the period, and the iteration. The relaxation coefficients are adjusted according to:

$$\omega^n(t) = \omega^{n-1}(t) \times \begin{cases} 0.5 \text{ if } \dfrac{p^*(t) - p^{n-1}(t)}{p^{n-1}(t) - p^{n-2}(t)} < 0 \\ 1.2 \text{ otherwise} \end{cases}$$

This "adaptive" adjustment formula decreases the relaxation coefficients if no oscillation is observed. The amount of increase or decrease in the relaxation coefficients specified in this formula were derived from experimentation on the model.

Similar relaxation methods are used to adjust the prices and quantities computed by other process relations. For simple processes, fixed relaxation coefficients are sufficient and the relaxation is performed on the "capital charge" or difference between price and variable costs as defined by relation (9). Thus, for a simple process

$$p^n(t) = \omega_p[p^*(t) - \phi^n(t)] + (1-\omega_p)[p^{n-1}(t) - \phi^{n-1}(t)] + \phi^{n-1}(t)$$

where $p^*(t) - \phi(t)$ is the capital charge. Relaxation is not applied directly to the price because the fuel cost portion of the price has already been relaxed lower in the network.

For the allocation process, the relaxation adjustments are applied to $q^n(t)$. The relaxation equation is

$$q^n(t) = \omega_a q^*(t) + (1-\omega_a)q^{n-1}(t)$$

Similarly, the output of secondary material processes is relaxed using fixed relaxation coefficients as follows:

$$\Pi^n(t) = \omega_s \Pi^*(t) + (1-\omega_s)\Pi^{n-1}(t)$$

Convergence of the algorithm is approximately measured by the sum of the residual error terms. For a resource process, the residual error term is as follows:

$$\hat{E} = \sum_{\text{all } t} \frac{[p^*(t) - p^n(t)]\, q^n(t)}{(1+r_{inf})^t}$$

where the error terms are weighted by quantity produced and deflated to constant dollars. These error terms are summed over all resource processes and then divided by

$$\tilde{E} = \sum_{\substack{\text{all } t \\ \text{processes}}} \frac{p^n(t)\, q^n(t)}{(1+r_{inf})^t}$$

to produce an error measure expressed as

$$\bar{E} = \frac{\hat{E}}{\tilde{E}}$$

Similar error measures are computed for other prices and quantities and the results are studied and used to improve relaxation coefficient settings or to adjust the parameters of the adaptive relaxation coefficient formula.

Computational Experience

The network iteration algorithm has been applied to a wide variety of problems. In the present version of the SRI-Gulf model, satsifactory convergence is achieved in 10 to 30 iterations for a sensitivity case (changes in model parameters from nominal settings) and 30 to 60 iterations for a nominal case beginning with poor initial estimates of prices and quantities. Generally, the number of iterations is not affected by the size of the model, but is strongly affected by the character of the process relations and the associated input parameters. For example, natural gas supply curves with rapidly changing slopes make the allocation of gas demand among gas supply regions more sensitive to changes in the algorithm and therefore tend to make convergence more difficult. In other applications of the same algorithm, convergence is generally achieved in 10 iterations or less, even with poor initial estimates, suggesting the possibility of improvements in the present algorithm.

Each iteration of the current model requires about 1.3 minutes of IBM 370/168 computer time and uses about 600,000 bytes of core storge. About 2700 processes are contained in the current model. Considerable savings in computer time and improved flexibility and ease of use are anticipated from a project presently underway to recode the model.

Uniqueness of Solutions

The model has been solved for many parameter sets and sensitivity cases. In more than three years of application of the model, there has been no evidence of multiple solutions. In general, the solutions to the model respond smoothly to changes in parameters, suggesting that the solution is unique. Some instability in the trajectories of prices and quantities has been experienced, but has always been confined to relatively small changes and usually can be eliminated by careful tuning of the relaxation coefficients in the algorithm.

No general theoretical conditions for uniqueness of the solutions to the model have been established. Clearly, such conditions would require restrictions on the process relations and, if these were easily understood, would provide additional clarification of the nature of the model's solutions. By drawing on available uniqueness conditions for solutions of systems of nonlinear equations or for general economic equilibrium theory, it is possible to state uniqueness conditions. Such conditions, however, are either so difficult to interpret or so restrictive as to be of little practical value. Further research on uniqueness of model solutions would be of value if it would lead to practical guidelines that would help the modeler to identify formulations where the solution is not unique.

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†$p^*(t)$ replaces the term $p(t)$ computed by the behavioral relations. This change in notation avoids having to use superscripts in the relations defined in the previous section.

VI. RELATION TO OTHER METHODOLOGIES

Other major energy models constructed in the last several years have used the full range of modeling methodologies including econometrics, mathematical programming, dynamic programming, control theory, general competitive equilibrium theory, and systems dynamics [12, 13, 14, 15, 16, 17, 18, 26]. Typically, demand-oriented models utilize econometric methods relying on continuous functional relations derived from historical data using statistical procedures. Supply-oriented models more often use the methods of mathematical programming with data developed subjectively from engineering estimates and data. Recently, the trend has been to link econometric energy demand models with mathematical programming energy supply models [14, 19]. Generalized equilibrium modeling, rather than linking models together, combines the basic concepts of all of the modeling methodologies in an integrated modeling framework or language.

The purpose of the following discussion is to provide the reader with additional insight into the generalized equilibrium methodology by identifying the basic concepts that characterize each of the major modeling methods and briefly indicating how each of these concepts is incorporated in the generalized equilibrium approach. For a detailed comparison of energy modeling methodologies, the reader is referred to reference [20].

Iterative Solution of Nonlinear, Simultaneous Equations

Any model constructed using the methodologies discussed in this report can be reformulated as a system of nonlinear, simultaneous equations. For example, the system of equations for an activity or optimization model is the set of necessary conditions for an optimum. Similarly, the system of equations for an econometric or general competitive equilibrium model is given by the demand and supply functions of the model. Systems of equations for static and dynamic models are similar except that the number of equations and unknowns in a dynamic model is proportional to the number of time periods in the model. A general system of nonlinear equations can be written as

$$f_i(x_1, \dots, x_n) = y: \text{for } i = 1, \dots, n$$

where x_j's are unknowns, and the y_i's are fixed parameters. In models where the system of equations has a serial structure of the form

$$f_i(x_1, \dots x_i) = y_i \qquad \text{for } i = 1, \dots, n$$

the solution of the equations can be achieved by solving the equations one by one in ascending order of i. In some models, the sets of equations and unknowns, such as those for a given time period, will be simultaneous but will also have a serial structure in relation to other sets. In general, however, optimization models, general competitive equilibrium models, and generalized equilibrium models are simultaneous and require iterative methods for solution.

A basic method of solving general, nonlinear systems of equations is known as the SOR or *successive over-relaxations* method [21]. The SOR method generates successive approximations to a solution according to the following adjustment process:

$$x_i^n = x_i^{n-1} + \omega(x_i - x_i^{n-1}) \qquad \text{for } i = 1, \dots, n$$

where n is the iteration index and the x_i's are the unknowns in the general system of nonlinear equations defined above.

The parameter ω is called the *relaxation coefficient*. For ω equal to unity the SOR method is equivalent to the Gauss-Seidel method of solving systems of equations [21]. Generally, in the SOR method ω is adjusted until satisfactory coverage is obtained or it is clear that the algorithm will not converge to a solution for the problem at hand.

A generalized equilibrium model is also easily interpreted as a system of nonlinear simultaneous equations. Furthermore, the relaxation techniques and iterative algorithm described in Section V for the SRI-Gulf model are clearly analogous to the SOR method. The SRI-Gulf algorithm is different in that many relaxation coefficients are used in the SRI-Gulf model and each is tied to specific equations or relations in the model. Another difference is that the network structure of the model allows process relations or equations to be expressed in a way that retains the natural structure of the problem and improves the insight-generating nature of the model. These two differences also greatly enhance the convergence of the SRI-Gulf algorithm over that of the similar SOR method, because insight into the solution process from the structure of the models is more easily used in setting the individual relation coef ficients, than would be the case if the general SOR method were directly applied to a system of equations.

More powerful solution methods, such as those related to Newton's method, use information about the derivatives of the equations to speed convergence. Such methods, however, require calculation or approximation of the matrix of first partial derivatives of each equation with respect to each unknown. The SRI-Gulf model is equivalent to about 100,000 equations and unknowns.* The matrix of first partial derivatives for such a model would require 10 billion elements, making impractical the direct application of such methods to large problems. However, more sophisticated algorithms that have been developed for solving systems of equations would be useful in small generalized equilibrium models or might be used on parts of a large model network that are particularly difficult to converge.

Econometric Methods

Econometric modeling methods are distinguished by their use of statistical methods to infer the parameters of a model from historical data. To facilitate statistical analysis, econometric models generally employ simplified, continuous models of consumer and producer behavior. These simplified functional relations, usually linear or linear in the logarithms of variables, tend to be useful approximations over only a small range about the historical operating point. When operation of the model's functions over wide ranges is necessary, as a result of uncertainty or long time horizons, such simplified functions often produce nonsensical results. Thus, a challenge to modelers is to develop practical modeling techniques that can utilize historical data while at the same time allowing for technical and other structural changes from historical trends, such as can be accomplished with optimization or activity analysis methods.

Econometric relations are easily incorporated in a generalized equilibrium model. For example, the end-use demand process relations and the allocation (market-share) process relations are continuous function models of individual and market behavior where much can be inferred from historical data. In the discussion of optimization methods which follows, the integration of econometric and optimization methods in generalized equilibrium modeling is discussed.

Optimization and Programming Methods

An optimization or mathematical programming model characterizes a system as a set of activities or processes where the allocation of resources and demand among the activities is determined by optimization of a single objective function. A major advantage of programming methods is that they force the modeler to be explicit about technological alternatives and physical flows and, as a result, programming models typically convey more physical insight than econometric models. Also, programming models, through their explicit representation of technological change and substitution, can provide for realistic model behavior over wide ranges of operating conditions and long time horizons. For this reason, linear programming is employed in a number of important energy models [12, 13, 14, 15,16] that are characterized by relatively-detailed energy supply sectors.

A limitation of linear programming and the models using linear programming is the restriction to linear functions. This limitation can be partially overcome by using piecewise linear and nonlinear programming methods, at the price of greater complexity. Another difficulty with linear programming is the *bang-bang* nature of its solutions. For example, an activity such as production of a fuel, that is slightly less costly than another fuel-producing activity, will be allocated all of the demand for that fuel. Additional constraints can subdue this *bang-bang* behavior only at the price of arbitrarily changing a model's solution.

A generalized equilibrium model combines the detailed activity modeling of programming methods with the continuous allocations of econometric analysis. For example, a conversion process in the SRI–Gulf model energy network can be viewed as an activity in the same sense that a mathematical programming model views an activity. However, in a generalized equilibrium model the selection among activities is accomplished by allocation processes that reflect market or decentralized decision-making rather than the single objective function of mathematical progrmaming. As a result, the multiple objective functions of the market can be made explicit in a generalized equilibrium model. Also, the continuous nature of the allocations produced in a generalized equilibrium model means that the *bang-bang* solutions of linear programming are replaced by a smooth response to relative prices of competing sources.

Generalized equilibrium models can be used as optimization models if desired, simply by replacing the multiple objective functions embedded in the individual conversion processes in the network by a common objective function, and by adjusting the parameters of the relations in the allocation processes to reflect cost minimization (use a large value for the market share parameter) combined with some changes in the relaxation methods. Used in this way, the network iteration algorithm will determine the mix of technologies, production quantities, and interregional flows that would minimize the cost of meeting a fixed demand or would maximize the sum of consumers' and producers' surplus.

When used for optimization, the methods of generalized equilibrium modeling resemble those of the generalized Lagrange multiplier method [22]. Using this method, the solution to an extremely general, constrained optimization problem is determined by iterative adjustment of the shadow prices (Lagrange multipliers). An even closer relationship between the generalized Lagrange multiplier method and generalized equilibrium modeling is obtained by reformulating the generalized Lagrange multiplier method so that it can be applied to unconstrained problems [7].

Dynamic programming and control theory models are also closely related to generalized equilibrium models. For those who are familiar with dynamic programming and control theory, the interpretation of these methods as Lagrange multiplier problems should be clear. Dynamic programming and control theory models are generally straightforward applications of the generalized Lagrange multiplier method to dynamic problems. While the mathematics of these methods is often complex, dynamic programming and control theory methods result in a set of equations that must be solved. These equations have a form similar to those of generalized equilibrium models. The difference is that dynamic programming and control theory models assume optimization of a single objective function whereas generalized equilibrium models can work with either a single objective function or multiple allocation models that are descriptive of decentralized or market decision-making processes.

General Competitive Equilibrium Models

General competitive equilibrium theory is concerned with the modeling of an idealized, decentralized economy [22, 23, 23, 25]. In a general competitive equilibrium model, consumer and producer behavior is described in terms of utility and production functions, and demand and supply functions for each commodity or material in the economy are derived from these utility and production functions assuming utility and profit maximization. The conditions for equilibrium are that the excess demands for each commodity be non-positive. Stated mathematically,

$$z_j(\underline{p}) \leqslant 0 \qquad \textit{all } \underline{j} \text{ commodities}$$

where

$$z_j(\underline{p}) = d_j(\underline{p}) - s_j(\underline{p})$$

$$\begin{pmatrix}\text{excess demand @}\\ \text{price vector } \underline{p}\end{pmatrix} = \begin{pmatrix}\text{demand @}\\ \text{price vector } \underline{p}\end{pmatrix} - \begin{pmatrix}\text{supply @}\\ \text{price vector } \underline{p}\end{pmatrix}$$

The equilibrium price vector, $\underline{p}^*$, satisfying these equilibrium conditions is determined by an iterative process that can be visualized as an auctioneer calling out tentative price vectors and then adjusting prices higher for those commodities with excess demand, and lowering prices for those commodities with negative excess demand (prices are restricted to be positive). In this way, the operation of a perfectly competitive economy is simulated.

General competitive equilibrium theory has stressed the study of existence and uniqueness conditions for an equilibrium solution and the stability of the solution. An algorithm based on extensions of the simplex method of linear programming has been developed for solution of very general problems [23]. The algorithm, however, appears to be limited to about forty equations.

Generalized equilibrium modeling as described in this paper differs from general competitive equilibrium models in that the market adjustment process is described in the process relations. For example, the effect on product prices of rapid increases in capacity is explicitly modeled in the SRI-Gulf model. The network iteration algorithm, unlike the auctioneer of general competitive equilibrium theory, finds a solution that incorporates the characteristics of the market adjustment processes described by the model relations as well as the supply and demand relations. The algorithm of a generalized equilibrium model need not bear any relation to actual market processes; it is simply a method of finding prices and quantities that satisfy the model relations.

Another difference between these methods is that the solution to a generalized equilibrium model represents a balance of forces in a descriptive, decentralized system rather than the idealized, decentralized system treated by general competitive equilibrium modeling. Except for this difference, and some important computational and model formulation differences, generalized equilibrium modeling is consistent with the basic concepts of general competitive equilibrium analysis.

Simulation Methods

Simulation methods model the dynamic evolution of a system by describing the physical processes and decision rules used within a system. Simulation methods allow the modeler great freedom in describing a system with relatively few restrictions on the form of the functional relationships. This ability to be descriptive is a major reason for the popularity of systems dynamics models among some modelers and users [26].

In modeling decision processes using a simulation methodology, explicit assumptions about the forecasting methods of decision-makers are usually made. Typically, these assumptions take the form of simple rules that extrapolate past trends in the model. However, in a

*The 100,000 equations and unknowns of the model result from about 2,700 processes times 17 time periods with approximately two equations and unknowns per process (a price and a quantity).

simulation model, prices and quantities within a given period typically are not determined simultaneously, but depend only on the prices and quantities in previous periods. As a result, the decision behavior modeled by a simulation model tends to be extremely myopic; no foresight is allowed. In some situations, such myopic behavior may be descriptive, but in most situations where decision-makers can anticipate changes in trends, a simulation model is not descriptive. Dynamic optimization, general competitive equilibrium, and generalized equilibrium models typically use future prices and quantities in computing present prices and quantities. Such forward-looking computations are important in computing investment decisions and economic rents, but if not used appropriately can overstate the forecasting ability of decision-makers.

In a generalized equilibrium model, the flexibility of simulation models is retained while allowing for a completely general dynamic structure. Rather than assuming either myopic or perfect forecasting, a generalized equilibrium model can be constructed with any degree of forecasting. For example, the behavioral lag relations of the SRI–Gulf model essentially interpolate between perfect and myopic forecasts. More general forecasting models are clearly possible within the basic structure of the model. Such models would be explicit about how expectations of the future are formed in market and other decision processes and would rely more heavily on studies of past decision-making behavior in order to be fully descriptive.

VII. OUTPUT OF THE SRI-GULF MODEL

The output of the SRI-Gulf model is documented in several publications oriented to specific applications [1, 3, 4, 5, 6]. For those readers who are not familiar with those publications, this section provides a very limited sample of the model's output.

Figures 22 through 25 show typical prices and quantities computed by the SRI-Gulf model. Figures 22 and 23 show the aggregated prices and quantities of primary resources that represent a dynamic supply and demand balance for the United States and Figures 24 and 25 give some of the more detailed prices and quantities. These prices and quantities are based on a nominal set of assumptions developed during 1976 [4]; sensitivity analysis to key assumptions has produced wide variations in the projections and this uncertainty has been accounted for in applying the model.

In Figure 22, the prices of primary resources are shown to increase as oil, gas, coal and nuclear resources are depleted (note that the prices are expressed in constant, 1975 dollars). In the near term, the prices of crude oil and natural gas increase rapidly. These price increases diminish the competitive position of oil and gas relative to other sources in the end-use markets, at which point other sources begin to replace oil and gas. These substitute fuels, nuclear, coal, oil shale, and synthetic gas and oil made from coal and oil shale, are relatively abundant in that the projected rate of depletion of the large deposits of these resources will not result in rapidly increasing prices. Since all energy products compete within the same residential, commercial, industrial and transportation markets, beyond the year 2000 the prices of crude oil and natural gas are competitive with the prices of products derived from nuclear, coal, and oil shale. These competitive end-use price levels reflect differences in the transportation costs and conversion costs of the fuels into products and costs of converting these products into useful energy. Thus the differences in the prices of primary resources shown in Figure 22 reflect both the market value of the resources relative to other resources and the costs of finding and extracting the resources.

Figure 23 shows the quantities of primary resources consistent with the prices of Figure 22. Coal and nuclear fuel are shown to grow faster than overall demand, while domestic and imported oil and gas decline in absolute quantities. Note that the quantity of a specific resource in Figure 23 is given by the vertical distance between the lines; thus, the top line in Figure 23 represents total primary energy production.

Figure 24 expands the detail on oil and gas production from Figure 23 and shows the large role played after the year 2000 by synthetic gas and oil from coal and shale.

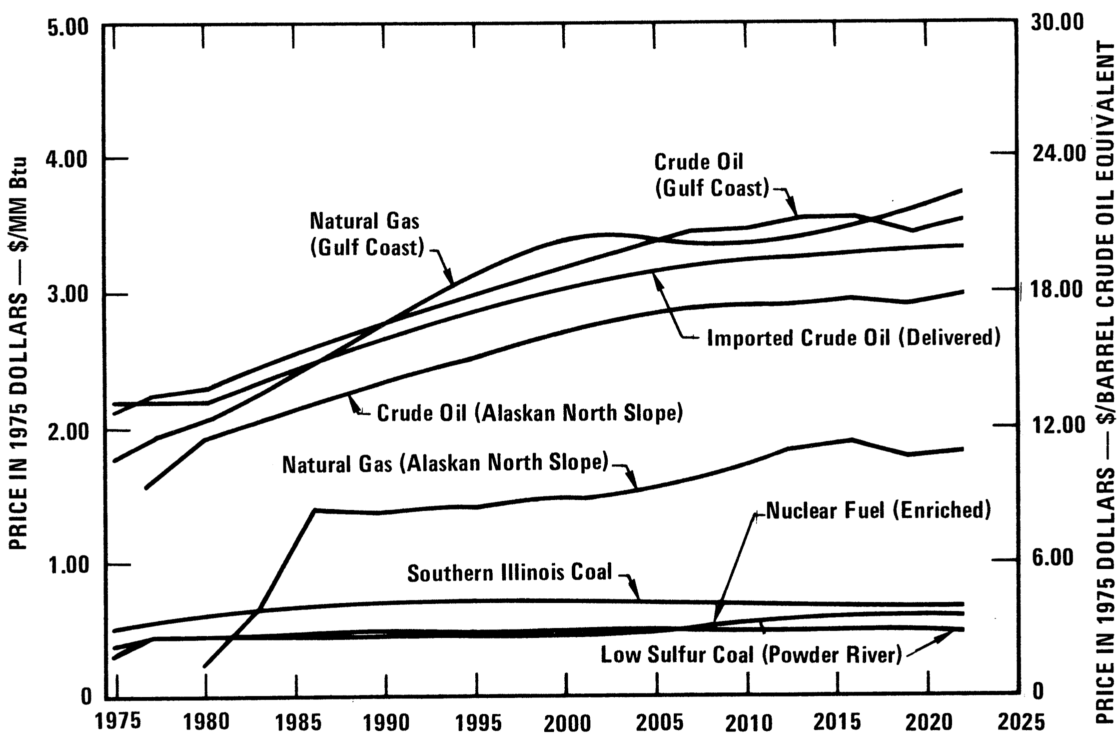

FIGURE 22. PRICES OF PRIMARY RESOURCES

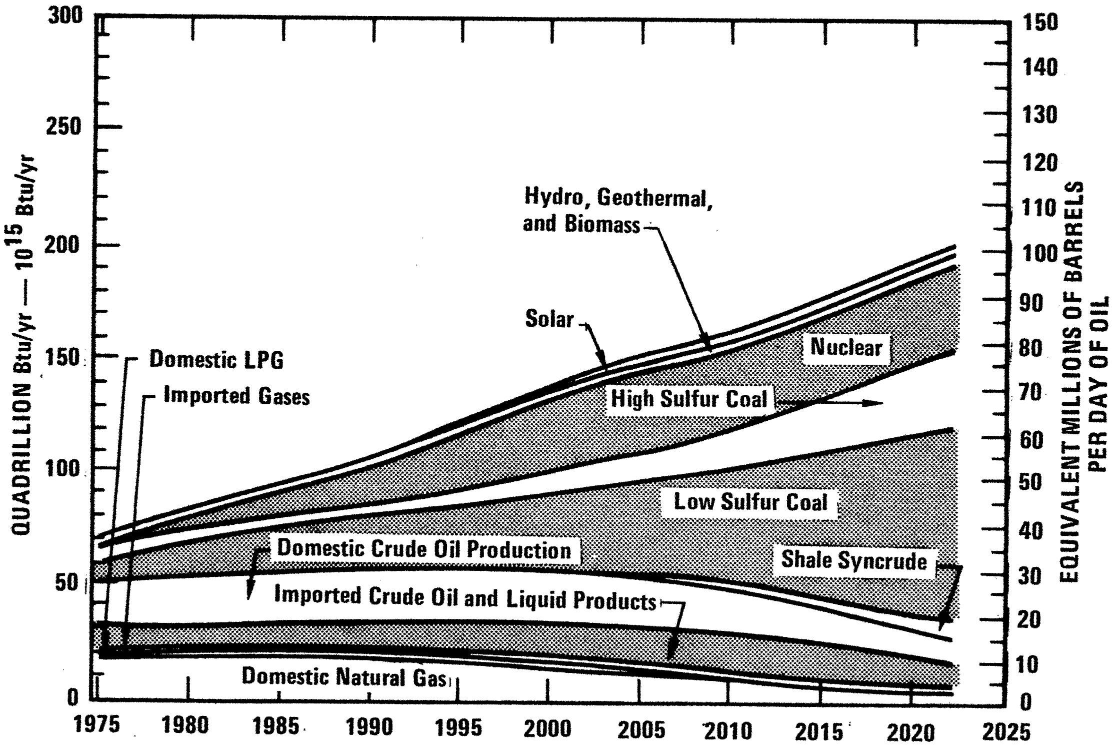

FIGURE 23. ANNUAL QUANTITIES OF PRIMARY RESOURCES

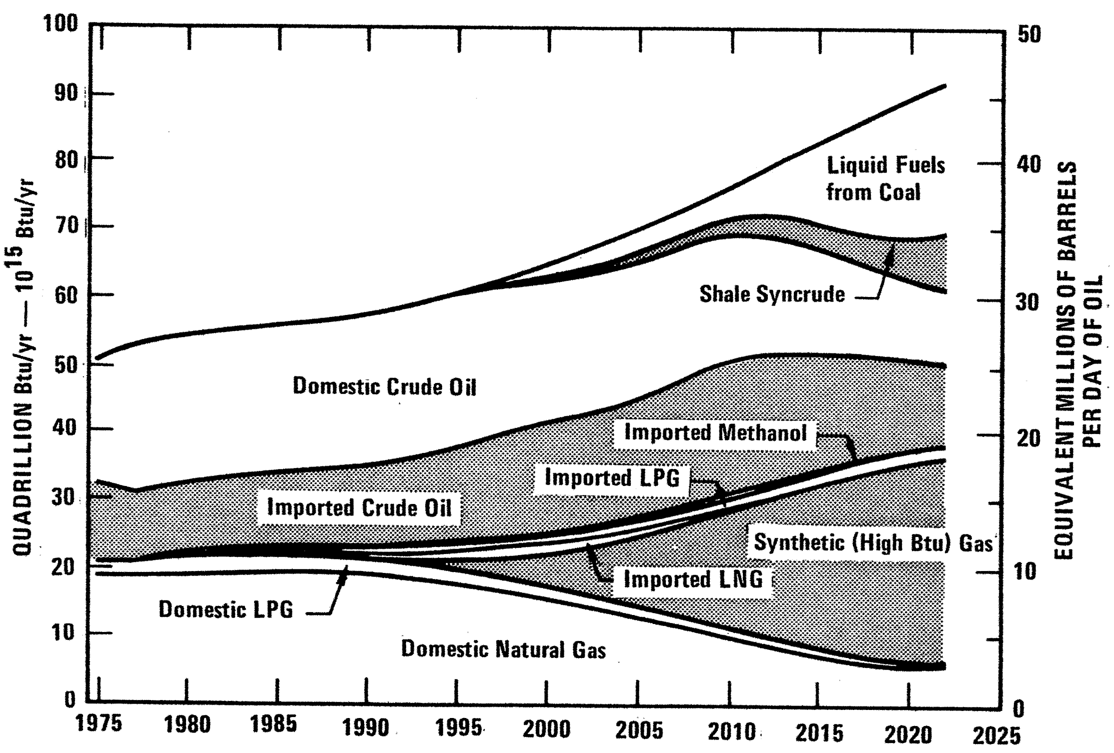

FIGURE 24. ANNUAL QUANTITIES OF LIQUIDS AND GASES

Figure 25 illustrates the sectoral detail of the model, showing the quantity of heat released as usable energy for residential and commercial space heating. By studying figures such as those shown here as well as other quantitative information, it is possible to work back through the model to understand how these projections are consistent with the physical and behavioral relations embedded in the processes and data assumptions of the model. Thus it is relatively easy to develop insight into the aggregated projections of the model by studying the detailed outputs of the model and the model relations and input data.

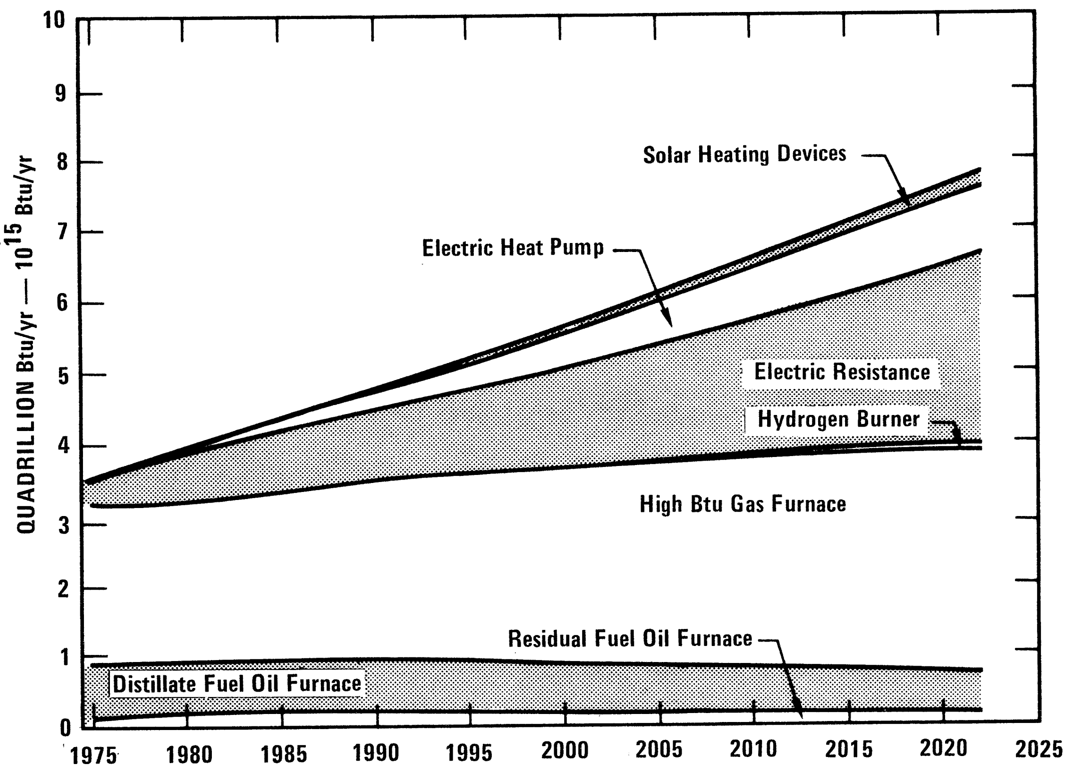

FIGURE 25. ANNUAL QUANTITIES OF RESIDENTIAL/COMMERCIAL USEABLE SPACE HEAT

VIII. DIRECTIONS FOR FUTURE RESEARCH

The methodology described in this paper has evolved from many years of repeated attempts to use quantitative models to provide insight to specific decision problems. The integration of modeling methodologies represented here was accomplished by practical necessity rather than theoretical design. The integration, however, is not complete, and the way is open for both theoretical and practical contributions in many areas.

On a practical level, work is now underway at Decision Focus, Inc. to develop improved, more flexible computer software to make the definitions of new models straightforward and to ease specific restrictions in the current computer language which have made difficult the full implementation of the generalized equlibrium modeling methodology and have slowed its application.

A particularly useful area for further applied and theoretical work is in the modeling of formation of expectations in economic markets. Those who have worked intensively with dynamic energy and economic models all seem to reach the same conclusion that modeling of the dynamics of the decision-making process is a very important, yet poorly understood aspect of dynamic models. With the flexibility provided by generalized equilibrium modeling, it is no longer necessary to restrict models to either perfect or myopic forecasting assumptions. Furthermore, although explicit representation of uncertainty in a generalized equilibrium model is not addressed in this report the basic theory has been developed and applied in other models [7, 8]. Thus, research and experimentation on models of the formation of expectations can now proceed more easily.

Another useful area of research is the development of improved techniques for combining historical data with other forms of information. The generalized equilibrium modeling methodology allows great freedom in specifying the relations or equations of a model. However, little has been done within the current model or by other researchers in the way of large-scale, statistical procedures to combine nonlinear regression analysis techniques with engineering data.

Applications of generalized equilibrium modeling concepts will undoubtedly expand as the approach becomes better understood and additional computer software becomes available. Previous applications of the methodology, in addition to the SRI-Gulf model, have included decomposition methods for modeling and planning capacity expansion in power systems under uncertainty [7], portfolio management under uncertainty [8], and decentralized corporate product and financial planning. Future applications must be focused on specific decision problems, but problems requiring regional or world models of entire economic, technical, environmental, and political systems are natural applications of generalized equilibrium modeling.

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